1. A spherical charge distribution is given by

\[ \rho = \rho_0 \left(1 - \frac{r^2}{a^2}\right), \quad \text{for} \quad r \leq a \]

\[ \rho = 0, \quad \text{for} \quad r > a \]

(a) calculate the total charge \( Q \). (7%)

(b) find the electric field intensity \( E \) outside the charge distribution. (7%)

2. Answer the following questions and justify your answer.

Suppose that in the Coulomb’s law the strength of the field is inverse cube proportional to distance \( \frac{1}{r^3} \).

(a) Is the field still conservative? (6%)

(b) Is Gauss’ law valid? (6%)

3. (a) A positive point charge \( Q \) is located at the distance \( d_1 \) and \( d_2 \), respectively, from two grounded perpendicular conducting half-planes, as shown in following figure. Determine the force on \( Q \) caused by the charge induced on the planes. (12%)

(b) An eccentric hole of radius \( a \) is bored parallel to the axis of a right circular cylinder of radius \( b \) (\( b > a \)). The two axes are at a distance of \( d \) apart. A current of \( I \) amperes flows in the cylinder as shown in the following figure. What is the magnetic field at the center of the hole? (12%)
4. A cladded-core optical fiber is shown below. Please find the maximum angle of incidence in terms of \( n_0, n_1 \) and \( n_2 \). (12 %)

![Optical fiber diagram]

5. Determine the characteristic resistance \( Z_0 \) of a lossless transmission line so that it will have a minimum standing-wave ratio for a load impedance 60+j80 \( \Omega \). (12 %)

6. A parallel plate waveguide composed of two perfectly conducting plates separated by a dielectric medium with constitutive parameters \( \varepsilon \) and \( \mu \) is shown below. Given \( E_y(y) = -\frac{\gamma}{h} A_n \cos \left( \frac{n\pi y}{b} \right) \).

(a) Please find the cut-off frequency of the dominant mode of this waveguide. (13 %)

(b) Find the surface charge densities of the upper and the lower plates. (13 %)