1. A system of linear equations is defined as \( A\vec{x} = \vec{b} \), where
\[
A = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 2 & -1 \\ 2 & 6 & 0 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}.
\]
(a) Solve the system of linear equations by Gauss-Jordan elimination. (6%)
(b) Find the dimension and a basis of the kernel of \( A \). (4%)

2. The manipulation of pictures in computer graphics is carried out using sequences of transformations. One common operation “translation” is defined by the following matrix addition:
\( T_1(\vec{u}) = \vec{u} + \vec{c} \) is a transformation from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \), where \( \vec{c} \) is a given vector in \( \mathbb{R}^2 \).
For this problem, assume \( \vec{c} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \).
(a) If \( \vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \), plot \( \vec{u} \) and \( T_1(\vec{u}) \) on the \( \mathbb{R}^2 \) plane. (2%)
(b) Prove that \( T_1 \) is NOT a linear transformation. (4%)
(c) Suppose, for any vector \( \vec{u} \) in \( \mathbb{R}^2 \), a third component of \( 1 \) is added to derive the vector \( \vec{v} \) in \( \mathbb{R}^3 \). (That is, if \( \vec{u} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \), \( \vec{v} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \).) Show that a linear transformation \( T_2(\vec{v}) = A\vec{v} \) from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \), can achieve the operation of translation, where \( A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \). (4%)

3. Consider a basis \( B \) of \( \mathbb{R}^3 \) consisting of the following vectors:
\[
\vec{v}_1 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -1 \\ \sqrt{2} \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.
\]
(a) Show that \( B \) is an orthonormal basis. (5%)
(b) If \( \vec{x} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} \), find its coordinates with respect to the basis \( B \). (5%)
4. Suppose a first-order polynomial is defined as follows:

\[ f(x) = \det \begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & 2 \\ 5 & 0 & 0 \\ x & 1 & 3 \end{bmatrix} = gx + h. \]

- a. Find the coefficient of \( x \), i.e., \( g \). (3%)
- b. Find the constant, i.e., \( h \). (4%)
- c. Find the value of \( x \) such that above 4x4 matrix used in defining the polynomial becomes non-invertible. (3%)

5. Consider the following 3x3 matrix: 
\[ A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 5 & -2 \\ -2 & 4 & -1 \end{bmatrix} \]

- a. Derive the characteristics polynomial of \( A \). (2%)
- b. Find the eigenvalue(s) of \( A \). (4%)
- c. Find the eigenspace and its basis for each eigenvalue. (4%)

6. Find \( \frac{dy}{dt} \) at \( t = 9 \) given that \( y = \frac{u + 2}{u - 1} \), \( u = (3s - 7)^2 \), \( s = \sqrt{t} \). (5%)

7. Evaluate \( \int x^2 e^{-x} dx \). (5%)

8. Find an equation for the plane tangent to the surface \( xy + yz + zx = 11 \) at the point \((1,2,3)\). (10%)

9. Use double integration to calculate the area of the region \( \Omega \) enclosed by \( y = x^2 \) and \( x + y = 2 \). (10%)

10. Use Green's theorem to evaluate \( \oint_C (3x^2 + y)dx + (2x + y^3)dy \) where \( C \) is the circle \( x^2 + y^2 = a^2 \). (10%)

11. Find the integral curves of the equation \( y' + 4y = 3e^{2x}y^2 \). (10%)