1. Consider the following LP problem,

Maximize \[ Z = 5x_1 + 3x_2 + 4x_3 \]

\[ \text{s.t. } \begin{align*}
  x_1 + x_2 + x_3 &\leq 10, \\
  4x_1 + 3x_2 + 2x_3 &\leq 24, \\
  4x_1 + 6x_2 + 3x_3 &\geq 36, \\
  x_1, x_2, x_3 &\geq 0.
\end{align*} \]

(1). Solve this problem by the Big-M method or the Two-Phase Method, step-by-step in tabular form. (10%)

(2). Construct the dual problem. (6%)

(3). Find the optimal solution of the dual problem from (1). (4%)

2. The FUN-TV Cable Company is in the process of planning a network for providing cable TV service to 8 new housing development areas. The cable system network is shown in Figure 1, and the number on each arc is the distance (in miles) between those two nodes.

(1). Find the shortest path from node 1 (the relay station) to node 9. (6%)

Figure 1. The cable system of FUN-TV Company

(2). Formulate a linear programming model for the shortest-path problem in (1). (Note: You don’t need to solve this LP problem.) (8%)

(3). Determine the links and the minimum cable miles while guaranteeing that all areas are connected (directly or indirectly) to the cable station (node 1). (6%)
3. (1). Solve the following constrained nonlinear programming (NLP) problem,

Maximize \( f(x) = 5x_1x_2 - 8x_3^2 \),

Subject to: \( 3x_1 + x_2 + 4x_3 = 12 \),
\( x_1, x_2, x_3 \in \mathbb{R} \).

(10%) 

(2). Show that the answer obtained in (1) is a local maximum by the sufficient condition given below. (10%)  

[※Note：呂秋文，”高等數學” p.447-448, 定理 8-14]  
給定一個 n 個變數 \((x_1, x_2, ..., x_n)\) 與 m 個等式限制條件(e.g., \(g_i(x) = b_i, \forall i = 1, 2, ..., m\))之非線性規劃問題，若存在向量 \(x^* \in \mathbb{R}^n, \lambda^* \in \mathbb{R}^m\)，使得本問題之拉氏函數之梯度等於 0，亦即 \(\nabla L(x^*, \lambda^*) = 0\);

令本問題之邊際賀氏(Bordered Hessian)行列式如下：

\[
\Delta_r(x^*) = \begin{vmatrix}
0 & \cdots & 0 & \frac{\partial g_1(x^*)}{\partial x_1} & \cdots & \frac{\partial g_1(x^*)}{\partial x_r} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots \\
0 & \cdots & 0 & \frac{\partial g_m(x^*)}{\partial x_1} & \cdots & \frac{\partial g_m(x^*)}{\partial x_r} \\
\frac{\partial g_1(x^*)}{\partial x_1} & \cdots & \frac{\partial g_m(x^*)}{\partial x_1} & L_{11} & \cdots & L_{1r} \\
\vdots & \ddots & \vdots & \ddots & \ddots & \ddots \\
\frac{\partial g_1(x^*)}{\partial x_r} & \cdots & \frac{\partial g_m(x^*)}{\partial x_r} & L_{r1} & \cdots & L_{rr}
\end{vmatrix},
\]

則(a). 若 \((-1)^r \Delta_r(x^*) > 0, \forall r = m + 1, ..., n\) 時，\(f(x^*)\) 為該非線性規劃問題之嚴格相對極小值。 (b). 若 \((-1)^r \Delta_r(x^*) > 0, \forall r = m + 1, ..., n\) 時，\(f(x^*)\) 為該非線性規劃問題之嚴格相對極大值。

4. Consider the stock market model and suppose that the stock’s going up tomorrow depends upon whether it increased today and yesterday. In particular, if the stock has increased for the past two days, it will increase tomorrow with probability 0.6. If the stock increased today but decreased yesterday, then it will increase tomorrow with probability 0.7. If the stock decreased today but increased yesterday, then it will increase tomorrow
with probability 0.4. Finally, if the stock decreased for the past two days, then it will increase tomorrow with probability 0.5.

If we define the states as follows,
State 0: The stock increased both today and yesterday.
State 1: The stock increased today and decreased yesterday.
State 2: The stock decreased today and increased yesterday.
State 3: The stock decreased both today and yesterday.
This leads to a four-state Markov chain.
(1). Construct the (one-step) transition matrix for this Markov chain. (4%)
(2). What is the probability that the stock will increase on Friday, given that it decreased on Tuesday and Wednesday? (8%)

[*Hint: You may need a two-step transition matrix to help you to answer this question.]
(3). After long time transition, what is the probability that the stock will increase on two consecutive days. (8%)

5. The coach of an age group swim team needs to assign swimmers to a 200-yard medley team to send to the Junior Olympics. Since most of his best swimmers are very fast in more than one stroke, it is not clear which swimmer should be assigned to each of the four strokes. The five fastest swimmers and the best time (in seconds) they have achieved in each of the strokes (for 50 yards) are

<table>
<thead>
<tr>
<th>Stroke</th>
<th>Carl</th>
<th>Chris</th>
<th>David</th>
<th>Tony</th>
<th>Ken</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backstroke</td>
<td>33.2</td>
<td>37.5</td>
<td>37.0</td>
<td>32.7</td>
<td>33.6</td>
</tr>
<tr>
<td>Breaststroke</td>
<td>43.1</td>
<td>34.6</td>
<td>41.0</td>
<td>33.0</td>
<td>39.6</td>
</tr>
<tr>
<td>Butterfly</td>
<td>33.1</td>
<td>30.2</td>
<td>28.4</td>
<td>38.7</td>
<td>32.4</td>
</tr>
<tr>
<td>Freestyle</td>
<td>29.0</td>
<td>29.3</td>
<td>26.1</td>
<td>28.7</td>
<td>28.1</td>
</tr>
</tbody>
</table>

The coach wishes to determine how to assign four swimmers to the four different strokes to minimize the sum of the corresponding best time.
(1). Define the decision variable, and formulate a LP model for this problem (not necessary in the form of assignment problem). (10%)
(2). Find the optimal assignment of four swimmers by the Hungarian Method. (10%)