1. Consider the following data: 5, 7, 4, 5, 21, 6. The median is
   (a) 5          (b) 5.5         (c) 6          (d) 8          (e) 21

2. Let \( Z \) have a standard normal distribution. Then \( \Pr(-2 < Z < 2) \) is approximately
   (a) 68.3%      (b) 80%        (c) 90%        (d) 95.5%      (e) 99.7%

3. Let \( U \) have a uniform distribution given by \( g(u) = 1/10, 0 < u < 10 \), zero elsewhere. Then the mean of \( U \) is
   (a) 5          (b) 10         (c) 15         (d) 20         (e) 25

4. Let \( X \) have an exponential distribution given by \( f(x) = \frac{1}{5} e^{-x/5}, x > 0 \), zero elsewhere. Then the variance of \( X \) is
   (a) 5          (b) 10         (c) 15         (d) 20         (e) 25

5. Let \( W \) have a chi-square distribution with 5 degrees of freedom. Then the variance of \( W \) is
   (a) 5          (b) 10         (c) 15         (d) 20         (e) 25

6. Let \( X \) and \( Y \) have a bivariate normal distribution with \( \sigma_X^2 = 25, \sigma_Y^2 = 36 \), and \( \rho = 0.5 \). Then the covariance of \( X \) and \( Y \) is
   (a) 5          (b) 10         (c) 15         (d) 20         (e) 25

7. Let \( X \) and \( Y \) have a bivariate normal distribution with \( \sigma_X^2 = 25, \sigma_Y^2 = 36 \), and \( \rho = 0.5 \). Then the variance of the conditional distribution of \( Y \), given \( X = x \), is
   (a) 6.25        (b) 9          (c) 12.5       (d) 18         (e) 27

8. Let \( \hat{p} \) for estimating the population proportion \( p \) be 0.5 based on a sample of size 100. Then the estimated standard error of \( \hat{p} \) is
   (a) 0.0025      (b) 0.005       (c) 0.025      (d) 0.05       (e) 0.25

9. Which of the following sampling distributions is used to test about the mean of a normal population when the variance is known?
   (a) the binomial distribution  (b) the standard normal distribution
   (c) the \( t \) distribution      (d) the beta distribution       (e) the \( F \) distribution
10. Which of the following sampling distributions is used to test about the mean of a normal population when the variance is unknown?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution

11. Which of the following sampling distributions is used to test if the variance of a normal population is equal to a given value?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution

12. Which of the following sampling distributions is needed for the matched pair comparison for means?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution

13. Which of the following sampling distributions is used to compare if the variances of two independent normal populations are equal?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution

14. Which of the following sampling distributions is used to test for goodness of fit?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution

15. Which of the following sampling distributions is used to test if a correlation coefficient is zero?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution

16. Which of the following sampling distributions is used to test the equality of several means in ANOVA problems?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution         (d) the χ² distribution          (e) the F distribution
二、複選題：(每小題正確答案至少一個，請選出正確的答案，須全部答對才能取得
該題分數，答錯不倒扣，每小題配分 4 分，合計 68 分。)

1. Let $S$ denote a sample space, and $A$ and $B$ be two events. Then
   (a) $P(S) = 1$
   (b) $P(S) = P(A) + P(B)$
   (c) $P(A \cap B) = P(A) P(B)$
   (d) $P(A \cap B) = P(A) P(A|B)$
   (e) $P(A \cup B) = P(A) + P(B) - P(B) P(A|B)$

2. Given the following probability distribution:

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.2</td>
</tr>
</tbody>
</table>

   Then (a) $\Pr(Y < 1) = 0.4$  (b) $\Pr(X < 1 \mid Y < 1) = 0.5$  (c) $\Pr(X = Y \mid Y \geq 1) = 0.5$
   (d) $\Pr(X > 1) = 0.35$  (e) $\Pr(Y > 1 \mid X > 1) = 0.4$

3. The variation of a distribution can be measured by
   (a) mean  (b) median  (c) range  (d) variance  (e) standard deviation

4. Let $X$ have a Bernoulli distribution with $p = 0.5$ and $Y$ have a Poisson distribution with
   mean $= 5$. Let $X$ and $Y$ be independent. Then
   (a) $E(X + Y) = 5.5$  (b) $E(10X - Y) = 0$  (c) $\Var(Y) = 25$
   (d) $\Var(Y - X) = 4.75$  (e) $\Var(2Y - 10X) = 45$

5. Let $X_1, X_2, \ldots, X_{50}$ constitute a random sample of size 50 from Bernoulli($p$) and $Y = \sum_{i=1}^{50} X_i$. Then
   (a) $Y$ is binomial $(50, p)$.
   (b) $\Pr(Y = 0) = (1 - p)^{50}$.
   (c) $\Pr(Y = 50) = (1 - p)^{50}$.
   (d) $\Var(Y) = 50p(1-p)$.
   (e) $\bar{X} = Y / 50$ is approximately $N(p, p(1-p))$. 
6. If $Z$ has a standard normal distribution, that is, $Z$ is $N(0,1)$, then

(a) $\Pr(Z > -1.2) = 0.5 + \Pr(Z > 1.2)$

(b) $\Pr(1 < Z < 1.2) = \Pr(Z < -1) - \Pr(Z < -1.2)$

(c) $\Pr(Z = 0) = 0.5$

(d) $E(Z^2) = 1$

(e) $\Var(Z) = 1$

7. Let $X_1, X_2, \ldots, X_n$ constitute a random sample of size $n$ from $N(\mu, \sigma^2)$. Let $\bar{X}$ and $S$ denote, respectively, the sample mean and sample standard deviation. Which of the following functions are statistics?

(a) $\bar{X}$

(b) $S / \bar{X}$

(c) $\sigma / \mu$

(d) $\sqrt{n} (\bar{X} - \mu) / S$

(e) $\Max\{X_1, X_2, \ldots, X_n\} - \Min\{X_1, X_2, \ldots, X_n\}$

8. Let $X_1, X_2, \ldots, X_{10}$ constitute a random sample of size 10 from $N(\mu, \sigma^2)$. Then

(a) $\sum_{i=1}^{10} X_i$ is $N(10\mu, \sigma^2)$.

(b) $\left(\sum_{i=1}^{10} X_i - 10\mu\right)^2 / \sigma^2$ is $\chi^2(1)$.

(c) $\sum_{i=1}^{10} (X_i - \mu)^2 / \sigma^2$ is $\chi^2(10)$.

(d) $\sum_{i=1}^{10} (X_i - \mu)^2 / \sigma^2$ is $\chi^2(9)$.

(e) $\sum_{i=1}^{10} (X_i - \bar{X})^2 / \sigma^2$ is $\chi^2(9)$.

9. Let $X_1, X_2, \ldots, X_n$ constitute a random sample of size $n$ from $N(\mu, \sigma^2)$. Let $\bar{X}$ and $S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1)$ denote, respectively, the sample mean and sample variance. Then

(a) $\bar{X} - \mu$ is $N(0, \sigma^2)$.

(b) $(X_2 - \mu) / \sigma$ is $N(0, 1)$.

(c) $\bar{X}$ and $S$ are independent.

(d) $\sqrt{n} (\bar{X} - \mu) / S$ has a $t$ distribution with $n$ degrees of freedom.

(e) $n(\bar{X} - \mu)^2 / S^2$ has an $F$ distribution with 1 and $(n-1)$ degrees of freedom.
10. Consider a random sample of size $n$ from a population distribution with mean $\mu$ and variance $\sigma^2$. According to the central limit theorem, when $n$ is sufficiently large,

(a) the population distribution is approximately normal with mean $\mu$ and variance $\sigma^2$.

(b) the population distribution is approximately $N(0, 1)$.

(c) the sample mean $\overline{X}$ is approximately $N(\mu, \sigma^2/n)$.

(d) $\sqrt{n} \left( \overline{X} - \mu \right) / \sigma$ is approximately $N(0, 1)$.

(e) the sample variance is approximately normal.

11. To construct a 95% confidence interval for a normal population mean $\mu$, based on a sample of size 20, when $\sigma^2$ is unknown, we need the information of

(a) sample mean

(b) sample median

(c) sample standard deviation

(d) the 95th percentile of the $t$ distribution with 19 degrees of freedom

(e) the 97.5th percentile of the $t$ distribution with 19 degrees of freedom

12. Select correct items about testing $H_0: \mu = \mu_0$ versus $H_1: \mu \neq \mu_0$ for a normal population mean $\mu$ at $\alpha$ level of significance:

(a) $Pr(\text{critical region} \mid H_0) = \alpha$.

(b) $Pr(\text{critical region} \mid H_1) = Pr(\text{Type II error})$.

(c) $Pr(\text{reject } H_0 \mid H_0) = p\text{-value}$.

(d) $Pr(\text{accept } H_0 \mid H_1) = \text{the power}$.

(e) reject $H_0$ if the $100(1 - \alpha)\%$ confidence interval for $\mu$ does not include $\mu_0$.

13. Select correct items about test of homogeneity and test of independence in two-way contingency table analysis:

(a) Both tests have the same null hypothesis.

(b) Both tests have the same test statistics.

(c) Both tests have the same sampling distribution.

(d) Both tests have the same sampling procedure.

(e) Both tests apply for large samples.

14. Which of the following sampling distributions are used to test if a regression coefficient is zero?

(a) the binomial distribution

(b) the standard normal distribution

(c) the $t$ distribution

(d) the $\chi^2$ distribution

(e) the $F$ distribution
15. Which of the following sampling distributions are used to test the equality of two population proportions?
   (a) the binomial distribution    (b) the standard normal distribution
   (c) the t distribution          (d) the $\chi^2$ distribution
   (e) the F distribution

16. Consider a multiple regression with $k$ ($>2$) independent variables based on $n$ observations. To test whether there is a regression relation between the dependent variable and the set of $k$ independent variables, use
   \[
   \begin{align*}
   \text{(a) } & \frac{SSR}{SSE} \\
   \text{(b) } & \frac{SSR/k}{SSE/(n-k-1)} \\
   \text{(c) } & \frac{R^2}{1-R^2} \\
   \text{(d) } & \frac{R^2/k}{(1-R^2)/(n-k-1)} \\
   \text{(e) } & MSE
   \end{align*}
   \]

17. According to the following regression output:

   **Analysis of Variance**
   
<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
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<td>55377.56221</td>
<td>18459.18740</td>
<td>6.865</td>
<td>0.0035</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>43022.18779</td>
<td>2688.88674</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Total</td>
<td>19</td>
<td>98399.75000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **Root MSE**  51.85448  R-square 0.5628
   **Dep Mean**  150.25000  Adj R-sq 0.4808
   **C.V.**      34.51213

   **Parameter Estimates**

   | Variable | DF | Parameter Estimate | Standard Error | T for H0: Parameter=0 | Prob > |T| |
   |----------|----|--------------------|----------------|-----------------------|--------|---|
   | INTERCEP | 1  | 67.037384          | 46.48740774    | 1.442                 | 0.1686 |
   | X1       | 1  | 3.626550           | 11.99909196    | 0.302                 | 0.7664 |
   | X2       | 1  | 2.322876           | 7.81454547     | 0.297                 | 0.7701 |
   | X3       | 1  | -0.779973          | 5.69767997     | -0.137                | 0.8928 |

   the overall $F$ test is significant, but the tests of individual regression coefficients are all insignificant. This may result from
   (a) outliers   (b) non-constant variance  (c) collinearity
   (d) autocorrelation (e) interaction