1. Suppose the unit-step response of a linear time-invariant system is
   \[ s(t) = (1 - 2e^{-2t} + e^{-t})1(t) \]
   where \( 1(t) \) is the unit-step function.

   (a) (5%) Determine the transfer function \( H(s) \) of the system.
   
   (b) (5%) Compute the response of the system to the sinusoidal input (assuming zero initial condition)
   \[ u(t) = (\sin t)1(t). \]

2. Consider the feedback system shown below.

   (a) (3%) Suppose the controller \( C(s) = 1 \). Determine the closed-loop transfer function \( H(s) \) from \( r \) to \( y \).
   
   (b) (6%) From the transfer function \( H(s) \) in (a), determine the damping ratio of the pair of complex-conjugate poles. What is roughly the overshoot (in %) of the step response of the system (taking into account the pair of real pole and zero)?
   
   (c) (3%) What is the velocity constant \( K_v \) of the system with \( C(s) = 1 \)?
   
   (d) (8%) Suppose now that you are asked to design a first-order controller of the form
   \[ C(s) = K \frac{s + z}{s + p} \]
   so that the velocity constant is increased by a factor of 2 (so the steady state performance is improved) while keeping the transient response roughly unchanged (i.e., the rise time and the overshoot should not change too much), what are the parameter values \( K, p, \) and \( z \) would you choose and why?
   
   (e) (2%) Is your design in (d) of lead- or lag-type?
   
   (f) (3%) What other effects do you expect your design in (d) to have on the feedback system’s step response?
3. Are the following True (T) or False (F)? Explain briefly. (A correct answer with no explanation will get no point.)

(a) (3%) If \( H(s) \) is the transfer function of a causal linear time-invariant system, then for any input \( u(t), 0 \leq t < \infty \), with Fourier transform \( U(j\omega), -\infty < \omega < \infty \), the Fourier transform of the output is given by \( H(j\omega)U(j\omega), -\infty < \omega < \infty \).

(b) (3%) If a feedback system, with impulse response \( h(t), 0 \leq t < \infty \), is stable and of type 1, then
\[
\int_0^\infty h(t)\,dt = 1.
\]

(c) (3%) A stable type 2 feedback system tracks unit-ramp input with no steady state error, hence its unit-step response \( s(t), 0 \leq t < \infty \), must have positive overshoot (i.e., there is \( t_0 > 0 \) so that \( s(t_0) > 1 \)).

(d) (3%) A lead controller with very large lead ratio is close to a PI controller.

(e) (3%) A feedback system with stable open-loop transfer function is always stable.

4. (30%) For a unit feedback system with an open-loop transfer function
\[
L(s) = \frac{K}{s(s+a)^2}, \text{ where } a > 0.
\]

(a) (6%) Plot the complete Nyquist diagram, including the asymptotes.

(b) (3%) If \( K = 2a^3 \), what happens to the closed-loop system?

(c) (3%) Why do you determine the closed-loop stability by the open-loop transfer function?

(d) (3%) What is the difference in the physical meaning, assuming the closed-loop system is stable for some \( K \), between the maximum overshoot of a unit-step response and the maximum of the magnitude frequency response of the closed-loop system?

(e) (6%) If the phase margin (PM) of the closed-loop system is 30°, what are the gain crossover frequency and the gain margin (GM) in dB?

(f) (3%) Write down a possible realization of a state space description of the controllable canonical form of the closed loop system,
\[
\frac{d}{dt} x(t) = Ax(t) + Bu(t),
\]
What are \( A \) and \( B \)?

(g) (6%) To solve the above issue in (b), we use the pole-placement design by state feedback \( u = -\beta x + r \) in (f). To speed up the system, we locate the desired dominant closed-loop poles by the following spec.: (1) the damping ratio is \( 1/\sqrt{2} \) and (2) the real part of the dominant pole is \( -a \). Let the remaining pole be \(-5a\), far away from the dominant poles, what is \( \beta \)?
5. (20 %) The Nichols chart of the open-loop transfer function $G(j\omega)/K$ of a unity-feedback control system is shown in the following.

(a) (4 %) If the input is $A \cos(\omega t)$ and the steady state output of the closed loop system is $B \cos(\omega t + \phi)$, what is $\omega$ when $B$ reach the maximum and what is the maximum?

(b) (5 %) Plot the corresponding Nyquist diagram for $\omega = 0 \sim \infty$. The key information must be in your plot.

(c) (4 %) Find the GM and PM, when $K = 1$.

(d) (3 %) Find the steady-state error of the system for a unit-step input.

(e) (4 %) Please observe very carefully and answer how many pole(s) and zero(s) at least does the system have based on your best knowledge? Why?