Problem 1 (16%)
What are the eigenvalues and eigenfunctions of the following problem?
\[ y'' + \lambda^2 y = 0 \]
\[ \text{st. } y(0) = 0 \quad y(L) = 0. \]
In addition, expand \( f(x) = x \) in terms of the above eigenfunctions in the interval \([0, L]\). Does this series converge uniformly? Why?

Problem 2 (17%)
Solve the following Helmholtz equation in 2-D Cartesian coordinates
\[ (\nabla^2 + k^2)u(x, y) = \delta(x - \frac{L}{3})\delta(y - \frac{H}{3}) \]
\[ \text{st. } \]
\[ \frac{\partial}{\partial x} u(0, y) = \frac{\partial}{\partial x} u(L, y) = \frac{\partial}{\partial y} u(x, 0) = \frac{\partial}{\partial y} u(x, H) = 0 \]
where \( k, L, \text{ and } H \) are constants and \( \delta \) is the Dirac delta function.

Problem 3 (16%)
(a) Show that \( FT\{x(-t)\} = X(-j\omega) = X^*(j\omega) \)

where \( j = \sqrt{-1}, \quad X(j\omega) = FT\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \) denotes the Fourier transform of a real function \( x(t) \).

(b) Express the Fourier transform of the cross-correlation function
\[ \int_{-\infty}^{\infty} x(t) y(t + \tau) dt \]
in terms of the Fourier transforms of two real functions \( x(t) \) and \( y(t) \).
Prob. 4 (17%)  

A, B and C are all 3D-space position vectors and \( \nabla = \left( \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \), show that

a) Show that \( A \times (B \times C) = (A \cdot C)B - (A \cdot B)C \).

b) Show that \( \nabla \times (\nabla \times C) = \nabla(\nabla \cdot C) - (\nabla \cdot \nabla)C \).

Prob. 5 (17%)  

Evaluate \( \int_C \frac{\cos az}{z^2 + 1} \, dz \), where contour C is shown as below and \( R \to \infty \).

![Diagram](diagram.png)

Prob. 6 (17%)  

Find the eigenvalues and eigenvectors of

\[
A = \begin{bmatrix}
\sigma & \omega \\
-\omega & \sigma
\end{bmatrix}
\]