1. (2%) What does the order $\Theta(f(n))$ mean?

2. (10%) Solve the following recurrence relations for the $\Theta$-order of $T(n)$:
   (a) $T(n)=T(\sqrt{n})+n^2$
   (b) $T(n)=T(n/2)+T(n/3)+T(n/6)+n$

3. (14%) Let $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{i=0}^{m} b_i x^i$. Assume that the inputs are given as $(a_0, a_1, ..., a_n)$ and $(b_0, b_1, ..., b_m)$.
   (a) Write an algorithm for creating linked-list data structures for storing $f(x)$ and $g(x)$.
   (b) Based on the linked lists, write an algorithm of $O(n+m)$ to add these two polynomials.
   (c) Based on the linked lists, write an algorithm of $O(n)$ multiplications to evaluate $f(w)$ for some given $w$.
   (d) Based on the linked lists, write an algorithm to compute $h(x)=f(x)\times g(x)$. What is the time complexity of your algorithm?

4. (14%) The data structure heap supports the operations of insert($x$) (insert number $x$), find-min (find the minimum number in the heap), extract-min (find the minimum number and delete it), etc.
   (a) What is a binomial tree?
   (b) How many binomial trees does an $n$-node binomial heap contain?
   (c) Build the binomial heap for performing the following operations in order:
      insert(3), insert(9), insert(2), insert(8), insert(1), insert(4), insert(10), insert(5),
      insert(13), extract-min, insert(6), insert(7), extract-min. Show the result after each operation.
   (d) What are the complexities of performing insert($x$), find-min, and extract-min, respectively?
5. IP Security (IPSec)
   (a) (5%) Why is IPSec necessary?
   (b) (5%) What are the benefits of IPSec?
   (c) (5%) Name two possible applications of IPSec.

6. Multi-protocol Label Switching (MPLS)
   (a) (5%) What is MPLS?
   (b) (5%) What are the benefits of MPLS?

7. The Web pages in a Web site have been simplified and organized in a degree-four tree structure.
   (i) The root node represents the home page. Each internal node represents a Web page with at most four reference links to other Web pages. All Web pages, except the home page, are referenced by only one Web page. The leaf node represents a Web page that does not have any reference links to other Web pages. A tree structure may be skewed.
   (ii) A reference path is defined as a list of Web pages traversed from the Home page (root node) to a leaf node according to the reference links. The length of a reference path is the number of Web pages contained in the reference path. A maximal reference path is a reference path that contains the maximal number of Web pages.
   (a) (5%) Assume that the length of a maximal reference path in a Web site (organized as a degree-four tree structure) is $H$. What is the maximal number of reference paths that the Web site may have?
   (b) (7%) Write a recursive algorithm to find the length of the maximal reference path in the Web site.
   (c) (3%) Analyze the time complexity of your algorithm.
8. Consider the following set of simplified requirements for a Web-log database that is used to keep track of Web-page access on the server log.

(i) Each Web-page is described by a title and an unique URL. A Web page may link to several other Web pages, and may be linked by several other Web pages.

(ii) The database stores each client's cookie (as an identifier) and IP address. A client may request Web-pages (by URL). The database keeps track the timestamp that the server received the request of a Web-page from a client, and the number of bytes served to fill the request. A Web-page may be requested by clients in different timestamps.

(a) (8%) Draw an ER (Entity-Relationship) schema diagram for this application. You need to clearly indicate the cardinality ratio (1:1, 1:N, or M:N) and participation constraints (total or partial) of each relationship. (State clearly any additional assumptions you make)

(b) (5%) Map the ER schema into the corresponding relational database schema diagram. Specify all the primary keys and foreign keys.

9. Assume there are five e-Services (PizzaOrder, ClothesSale, HouseRental, CarRental, MovieRental) and seven e-Service providers (e-Com1, e-Com2, e-Com3, e-Com4, e-Com5, e-Com6, e-Com7) available on the Internet. Each e-Service provider is capable of providing the five e-Services.

(a) (3%) Suppose that there is no limitation on the number of e-Services that each e-Service provider can serve. What is the number of ways to select the e-Service providers to serve the five e-Services?

(b) (4%) Suppose that each e-Service provider can serve at most one e-Service. What is the number of ways to select the e-Service providers to serve the five e-Services?