A. 資料結構

1. (8%) Given a binary tree as the figure follows, (a) draw the memory representation of a threaded tree corresponding to the figure, (b) Write an algorithm for traversing the above threaded binary tree in preorder.

![Binary Tree Diagram]

2. (8%) Given an AVL tree, write an algorithm to list the nodes in ascending order of node.index data members. What is the time complexity of your algorithm if the tree contains n nodes.

3. (9%) Explain the following terms: (a) 2-3-4 trees (b) B-trees (c) Digital Search trees.

4. (13%) Given the Fibonacci number as
   
   1 2 3 5 8 13 21 34 …
   
   where the next Fibonacci number will be the sum of its previous two Fibonacci numbers. For example, number 8 will be the sum of the numbers of 3 and 5, the number of 34 will be the sum of the numbers of 13 and 21. Likewise, the next new Fibonacci number in the above example will be 55 which is the sum of 21 and 34.
   
   a) Write a recursive program to generate the Fibonacci number N.
   b) Write a non-recursive program using the loop structure to generate the Fibonacci number N.
   c) Discuss the execution efficiency of these two programs.

5. (6%) Given the following function definitions:
   
   create -> list
   cons (list, element) -> list
   head (list) -> element
   length (list) -> integer
   tail (list) -> list

   For the following list expression,
   Cons (cons (tail (cons ([], 5)), 7), 9)
   a) use link-list structure to represent the above expression
   b) evaluate the above expression

6. (6%) Given the following abstract data types definition

   class temperature
   {
      private:
      float highTemp, lowTemp;
      public:
      Temperature (float h, float l);
      Void updateTemp (float temp);
      Float GetHighTemp (void) const;
      Float GetLowTemp (void) const;
Temperature :: Temperature (float h, float l): highTemp (h) lowTemp (l) 
Void Temperature :: updateTemp (float temp)
    if (temp > highTemp) highTemp = temp;
    else if (temp < lowTemp) lowTemp = temp;

Float Temperature :: GetHighTemp (void) const
    return highTemp;
Float Temperature :: GetLowTemp (void) const
    return lowTemp;

Suppose the main program has two statements
    Temperature Awater(212, 32);
    Temperature Bwater(100, 0);

a) evaluate the operation of Awater.GetHighTemp and Bwater.GetLowTemp
b) Can one access the variables highTemp and lowTemp besides the way of

B. 計算法

7. (12%) Let $T(n) = \Theta(f(n))$. Derive $f(n)$ in the simplest formula for each of the following $T(n)$.
   a. $T(n) = 4T(n/2) + n^2 \log n, T(c) = c, \text{if } c < 2.$
   b. $T(n) = 4T(n/2) + n^2; T(c) = c, \text{if } c < 2.$
   c. $T(n) = 3T(n/2) + n^2 \log n, T(c) = c, \text{if } c < 2.$
   d. $T(n) = 3T(n/2) + n^2; T(c) = c, \text{if } c < 2.$
   e. $T(n) = 1 \times n + 2 \times (n-1) + ... + (n-1) \times 2 + n \times 1.$
   f. $T(n) = n + n/2^2 + n/3^2 + ... + n/(n(n))^2.$

8. (8%) Given positive constants $c', c_1, c_2, ..., \text{and } c_k$, assume that $T(n) \leq T(c_1 n) + T(c_2 n) + ... + T(c_k n) + c' n$ and $c_1 + c_2 + ... + c_k < 1$. Prove $T(n) = O(n)$.

9. (6%) Describe an algorithm to solve the product of two polynomial functions, $P(x) = p_0 x^0 + p_1 x^1 + p_2 x^3$ and $Q(x) = q_0 x^0 + q_1 x^1 + q_2 x^2$, in time complexity $O(n \log n)$. Hint: you can directly base your algorithm on FFT algorithms.

10. (8%) The longest increasing subsequence problem is: Given a sequence of distinct integers $x_1, x_2, ..., x_n$, find a longest increasing subsequence, $x_{i_1}, x_{i_2}, ..., x_{i_k}$ with $1 \leq i_1 < i_2 < ... < i_k \leq n$ such that $x_{i_j} < x_{i_{j+1}}$ for all $j$. Design an efficient algorithm to solve this problem and illustrate your algorithm by the following example, (66, 92, 123, 31, 83, 53, 48, 17, 9, 57, 75).

11. (8%) Prove that the time complexities for matrix squaring and symmetric matrix multiplication are the same. Hint: matrix squaring is to derive $A \times A$, where $A$ is an $n \times n$ matrix; symmetric matrix multiplication is to derive $A \times B$, where both $A$ and $B$ are $n \times n$ symmetric matrices.

12. (8%) Prove that the clique problem is NP-complete. Hint: reduce SAT to the problem and use one example to illustrate.