1. (10%) Suppose the unit-step response of a system is 

\[ s(t) = (1 - 3e^{-2t} + 2e^{-3t})1(t) \]

Compute the system's response, \( y(t) \), to the input \( u(t) = \cos(t), \ t \geq 0 \).

2. (20%) The Bode plot of the loop transfer function of a unity feedback system is shown below, where the magnitude is in dB, phase in degrees, and frequency in rad/sec. Answer the questions below. Please show the calculations you use to arrive at each answer or explain your reasoning briefly.

![Bode plot](image)

(a) (3%) Determine the gain margin GM and phase margin PM of the system.

(b) (3%) Roughly, what is the steady state error of the step response of the feedback system?

(c) (3%) Roughly, what is the rise time of the step response of the feedback system?

(d) (3%) Roughly, what is the peak overshoot of the step response of the feedback system?

(e) (8%) Suppose you are asked to use controller \( C(s) \) of the form

\[ C(s) = K \frac{s + z}{s + p} \]

in order to increase the error constant by a factor of 5 (to improve the steady state response) while maintaining the phase margin, roughly what values of \( K \), \( z \), and \( p \) would you choose?
3. (10%) Show that the unit-step response of a type 2 (with respect to reference input) unity-feedback system always has positive overshoot \( i.e., M_p > 0 \).

4. (10%) Consider the system given by

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-1 & 1 \\
-0.5 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
1 \\
0.5
\end{bmatrix} u,
\]

\[
y = \begin{bmatrix}
1 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

Determine the initial condition \( x(0) \) so that for the input \( u(t) = e^{-at} \), \( t \geq 0 \), the output is \( y(t) = ke^{-at} \), \( t \geq 0 \), where \( k \) is a real constant. Determine \( k \) for the present case. Note that the output \( y(t) \) does not have any transient components.

5. (24 %) Let a unit-feedback system with the open-loop transfer function

\[
G(s) = \frac{K(s^2 + z^2)}{s(s^2 + p^2)}, \quad \text{where } p, z > 0.
\]

(a) (6 %) Plot the root-loci for \(-\infty < K < \infty\) in case of \( p > z \) and \( z > p \), respectively. Check your conclusion by Routh’s criterion.

(b) (8 %) If we want the zero steady state error for a ramp input, you can simply add one more pole at the origin, \( i.e., G_1(s) = G(s)/s \). Consider the case of \( K > 0 \) only. Plot the root loci for \( p > z \) and \( z > p \), respectively.

(c) (10 %) Consider the system in part (c). Add one more zero to the system, that is \( G_1(s) = \frac{(s + z_1)}{s} G(s) \), where \( z_1 > 0 \). Consider the case of \( K > 0 \) only. Plot the root loci for \( p > z \) and \( z > p \), respectively.
6. (26 %) Consider the following feedback system with the plant \( G_p(s) \) and the controller \( G_c(s) \).

\[
\begin{array}{c}
\text{r} \quad + \quad e \\
\text{ } \quad \downarrow \\
G_c(s) \quad \downarrow \quad G_p(s) \\
\text{ } \quad \parallel \\
\text{ } \quad \downarrow \\
\text{y} \\
\end{array}
\]

(a) (4 %) The asymptotic magnitude response of the plant is shown in the right. Determine the transfer function of \( G_p(s) \) from \( |G_p(j\omega)| \) the response.

(b) (3 %) If the maximum overshoot is limited to be less than 5\% for the step input, please design a P control to achieve the goal so that the target system will have maximum overshoot 4.3\%.

(c) (10 %) If the poles of the second order closed-loop system locate at the four possible points A, B, C and D in the s-plane shown in the right figure, you, as an expert control engineer, are asked to pick up the desired poles to satisfy the following specifications. (No credit can be given without explanation.)

i. (2 %) the fastest settle time
ii. (2 %) the smallest maximum overshoot
iii. (2 %) the biggest bandwidth
iv. (2 %) the smallest oscillation
v. (2 %) the slowest response and biggest maximum overshoot

(d) (3 %) Since the plant obtained in part (b) can't simultaneously satisfy both of the spec. of the settling time \( t_s \leq 0.2 \text{ sec} \) by the approximation of \( t_s \approx \frac{4}{\zeta \omega_n} \) and maximum overshoot \( \leq 5\% \), we can add one open-loop zero to shift to meet the requirement. Why?

(e) (6 %) Design the controller \( G_c(s) = K(s + z) \) to meet the above requirements by choosing the damping ratio \( \zeta = \frac{1}{\sqrt{2}} \)?