1. Find the general solution of the following differential equations:
   i) \( xy' = y^2 - y \)  
   (10 points)
   ii) \( y'' - y' + 2y = e^x \sin x \)  
   (10 points)

2. A temperature distribution \( T(x,y) \) at steady state satisfies the Laplace equation

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0
\]

If the boundary conditions are given as:

\[
T(x,0) = 0, \quad T(x,h) = f(x) \\
T(0,y) = 0, \quad \frac{\partial T(w,y)}{\partial x} = c
\]

Solve \( T(x,y) \) for i) \( c = 0 \) (10 points), ii) \( c \neq 0 \) (10 points).

3. (a) The points \( A(1,-2,1), B(0,1,6) \) and \( C(-3,4,-2) \) form a triangle. Find the angle between the line \( AB \) and the line from \( A \) to the midpoint of the line \( BC \).  
   (5 points)
(b) Find the equation of a plane passing through \((-6, 1, 1)\) and perpendicular to \(-2i + 4j + k\).  
   (5 points)

4. (a) Given \( B = \begin{bmatrix} 12 & 11 & -32 \\ -5 & 9 & 30 \\ 32 & -18 & 15 \end{bmatrix} \), write \( B \) as a sum of a symmetric and a skew-symmetric matrix.  
   (10 points)
(b) Prove that \( B^TB \) is symmetric.  
   (10 points)

5. Use the Laplace transform method to solve the following ordinary differential equation,

\[
x'' + x' + tx = 0 \quad (0 \leq t \leq \infty) \quad x(0) = 1, \ x'(0) = 0
\]

where prime denotes the differentiation with respect to \( t \).  

[Note: \( L[J_0(t)] = \frac{1}{\sqrt{s^2 + 1}} \) \( (s > 0) \), where \( J_0(t) \) is the Bessel function of the first kind.]

\( L: \) Laplace Transform; \( J_0(0) = 1 \)

6. (a). Solve the Fourier series representation of the function,

\[
f(x) = x^2 \quad -\pi \leq x \leq \pi
\]

(b) If the Riemann zeta function, \( \zeta(p) \), is defined by

\[
\zeta(p) = \sum_{n=1}^{\infty} \frac{1}{n^p}, \quad p > 1
\]

Based on the results of (a), determine the value of \( \zeta(2) \).  
   (5 points)