1. (12%) Solve
\[
\frac{d^2 y}{dx^2} + 4y = \cos 2x + \cos 4x
\]

2. (11%)
   (a) (6%) Find the following Laplace transform
   \[L\{t^2 \cos \omega t\}\]
   (b) (5%) Find the following inverse Laplace transform
   \[L^{-1}\left\{\frac{6s - 4}{s^2 - 4s - 20}\right\}\]

3. (10%) Find the Fourier series of the following periodic function \(f(t)\)
\[
f(t) = \begin{cases} 
1 + t^2, & 0 < t < 1 \\
3 - t, & 1 < t < 2 
\end{cases}
\]
\[f(t + 2) = f(t)\]

4. (13%) Prove that \(\nabla \phi\) is a vector perpendicular to the surface \(\phi(x, y, z) = c\), where \(c\) is a constant.

5. (20%)
   (a) (10%) Solve the linear system
   \[
   \begin{align*}
   2x + y + 2z + w &= 6 \\
   6x - 6y + 6z + 12w &= 36 \\
   4x + 3y + 3z - 3w &= -1 \\
   2x + 2y - z + w &= 10
   \end{align*}
   \]
   (b) (10%) Determine the eigenvalues and eigenvectors of
   \[
   A = \begin{pmatrix}
   \cos \theta & -\sin \theta \\
   \sin \theta & \cos \theta
   \end{pmatrix}
   \]
6. (10%) Determine the radius of convergence of the following cases:
   (a) \( \sum_{n=0}^{\infty} n!x^n \),
   (b) \( \frac{1}{1-x} \),
   (c) \( e^x \).

7. (10%) Determine the all cube roots of 27.
   (a) Evaluate \( \int (1-z)dz \), where \( C \) is given by \( z(t) = t - it \), \( t \) varying from 0 to 1.

8. (14%) The vibration of a stretched, flexible string problem:
   An elastic string, stretched under a tension T between two points on the axis. The weight of the string per unit length after it is stretched we suppose to be a known function \( w(x) \). Besides the elastic and inertia forces inherent in the system, the string may also be acted upon by a distributed load \( f(x,y,\dot{y},t) \). We assume that
   1. The motion takes place entirely in one plan, and in this plan each particle moves at right angle to the equilibrium position of the string.
   2. The deflection of the string during the motion is so small that the resulting change in the length of the string has no effect on the tension T.
   3. The string is perfectly flexible, i.e., can transmit force only in the direction of its length.
   4. The slope of deflection curve of the string is at all points and at all times so small.
   
   (a) Derive the one dimension wave partial differential equation
   \[ \frac{\partial^2 y}{\partial t^2} = \frac{Tg}{w(x)} \frac{\partial^2 y}{\partial x^2} + \frac{g}{w(x)} f(x,y,\dot{y},t), \]
   where \( g \) is acceleration of gravity.
   
   (b) Please check the dimensions (units) of three terms in (a).