25% 1. A 30-eV electron is incident on a barrier whose cross-section is a square of height $V_0=40$ eV. 1a) Write down the Schrödinger equation for the three regions. 1b) Solve the equations. 1c) Calculate the probability that the electron will tunnel through the barrier if its thickness is 1 nm ($10^{-9}$ m).

15% 2. According to our present understanding, Helium is formed in the sun (formed mainly with a plasma of protons and electrons) in a cycle of reactions, called the proton-proton cycle. The overview of the cycle is as following: $4\, ^1\text{H} \rightarrow \, ^2\text{He} + 2\, e^+ + \nu + 25$ MeV. Assuming that the average radiation power output of the Sun is $9 \times 10^{26}$ W. Use Stefan's law, $R_T = \sigma T^4$, $\sigma = 5.67 \times 10^{-8}$ W/m$^2$K$^4$, $R_T$ is the radiancy of a black body, i.e. the total energy emitted from its surface per unit area per unit time at temperature T. Take the sun's diameter to be $1.4 \times 10^9$ m and the surface temperature of the sun to be 5700 K. The rest mass $M_0$ of the Sun is about $2 \times 10^{30}$ kg.
2a) Determine the number of $^4\text{He}$ produced per second.
2b) The rest mass of the Sun is reduced by $\Delta M$ in 3 billion years ($\approx 10^{17}$ s). Calculate $\Delta M/M_0$.

15% 3. A γ ray creates an electron-positron pair. Show directly that, without the presence of a third body to take up some of the momentum, energy and momentum before and after the pair production cannot both be preserved.

10% 4. The frequency of photon that causes the $v=0$ to $v=1$ transition in the CO molecule is $6.42 \times 10^{13}$ Hz. Determine the ratio $n(v_1)/n(v_0)$ of the number of molecules in the first and lowest vibrational states.

15% 5. The CO molecule has moment of inertia I. The $J=0$ to $J=1$ rotational transition of the CO molecule occurs at $1.15 \times 10^{11}$ Hz. Calculate 5a) the moment of inertial of the molecule and 5b) the bond length of the molecule.

20% 6. Using the Bohr's Model, derive the quantization of 6a) the total energy of a hydrogen atom $E_n$, $n=1,2,3,...$ the principal quantum number 6b) and the orbital magnetic dipole moment $\mu_\ell$, $\ell$ is the azimuth quantum number 6c) If a homogeneous magnetic field $B$ is applied, predict the total energy of a hydrogen atom for different principal quantum number $n$ and magnetic quantum number $m_\ell$. 