1. (10%) Consider the system with input $r(t)$, output $y(t)$, and transfer function

$$G(s) = \frac{s + 8}{(s + 2)(s + 4)}.$$ 

(a) Find the steady-state response due to $r(t) = 2 + 3t$.

(b) Compute the steady-state error between $r(t)$ and $y(t)$.

2. (8%) Consider a linear system. Its zero-state responses due to $u_1$ and $u_2$ are shown in Figure P2 (a).

(a) Is the system time-invariant?

(b) Find the zero-state response due to the input $u_3$ shown in Figure P2 (b).

![Figure P2 (a)](image1)

![Figure P2 (b)](image2)

Figure P2 (a)  Figure P2 (b)

3. (12%) The schematic diagram of a motor-load system is shown in Figure P3. The following parameters are defined: $T_m(t)$ is the motor torque, $\omega_m(t)$ the motor velocity, $\theta_m(t)$ the motor displacement, $\omega_L(t)$, the load velocity, $\theta_L(t)$ the load displacement, $K$ the torsional spring constant, $J_m$ the motor inertia, $B_m$ the motor viscous-friction coefficient, and $B_L$ the load viscous-friction coefficient.

(a) Write the torque equations of the system.

(b) Find the transfer functions $\Theta_m(s)/T_m(s)$ and $\Theta_m(s)/T_m(s)$.

![Figure P3](image3)
4. (16%) Consider a unity feedback system. The transfer function of the closed-loop system is \( T(s) = \frac{KG(s)}{1 + KG(s)} \) with
\[
G(s) = \frac{1}{s(s + 3)[(s + 3)^2 + 7.425^2]}.
\]
The root locus of \( G(s) \) is shown in Fig. P4.

(a) Calculate the center of the asymptotes for the root locus and their leaving angles.
(b) Calculate the departure angle from the pole \( p_1 = -3 + j7.425 \) in the root locus.
(c) Find the value of \( K \) for which the main (closest to the origin) complex poles of the closed-loop system have the damping ratio of 0.5. (Hint: From the root locus, it is already obtained that the closed-loop complex poles with damping ratio of 0.5 are \( s = -1.27 \pm j2.2 \).)
(d) Determine the position-error constant and the velocity-error constant.

![Figure P4](image)

5. (8%) Draw the Nyquist plot of \( G(s) \),
\[
G(s) = \frac{1}{s^2 + 6^2},
\]
and determine the unstable region for the gain \( K \) of a unity feedback system whose transfer function is \( T(s) = \frac{KG(s)}{1 + KG(s)} \).

6. (16%) Consider the plant
\[
G(s) = \frac{1}{(s/0.5 + 1)(s+1)(s/2+1)}.
\]
A unit feedback system is constructed with a lead compensator \( KD(s) \), where
\[
D(s) = \frac{(Ts + 1)}{(\alpha Ts + 1)} \text{ and } \alpha < 1.
\]
(a) To make the steady-state error to a unit step input be 1/11, we need \( K = 10 \). Please
write out the \textbf{method} to get the value of $K$. 

(b) The Bode plot of the plant $G(s)$ is shown in Fig.P6. Find the \textbf{phase margin} (PM) and the \textbf{crossover frequency} ($\omega_c$) for $KG(s)$ with $K=10$.

(c) Suppose that we want the phase margin of the compensated system $KD(s)G(s)$ to be $25^\circ$ and assign a small extra margin $8^\circ$. Please calculate the required \textbf{maximum phase} ($\phi_{\text{max}}$) of the lead compensator $D(s)$, and then $|D(j\omega_{\text{max}})|$.

(Hint: $\alpha = (1 - \sin \phi_{\text{max}})/(1 + \sin \phi_{\text{max}})$ and $\omega_{\text{max}} = 1/(T \sqrt{\alpha})$.)

(d) Let $\omega_{\text{max}}$ be the frequency where $\angle D(j\omega_{\text{max}}) = \phi_{\text{max}}$, and new $\omega_{c}$ be the crossover frequency of $KD(s)G(s)$.

Assume $|KD(j\omega_{c})G(j\omega_{c})| = |KD(j\omega_{\text{max}})G(j\omega_{\text{max}})|$ to find the approximated $\omega_{\text{max}}$ from the Bode plot of $G(s)$.

(e) Finally, find the required \textbf{lead compensator} $D(s)$.
7. (30%) Consider the following control system:

\[
\begin{align*}
&\text{Compensator} \\
&G(s) = \frac{1}{(s+2)(s-2)} \\
&H(s)
\end{align*}
\]

Let \( H(s) = 1 \). Design a compensator such that the following specifications are met:
\[
\zeta = 0.5 \quad \text{and} \quad \omega_n = 2.
\]

(a) (4%) Let the compensator be \( C_1(s) = \frac{K_1(s+2)}{s+p} \). What is the compensator \( C_1(s) \)?

(b) (4%) What is the position-error constant \( K_p \) of the system with the compensator \( C_1(s) \)? What is the physical meaning if \( K_p < -1 \)?

(c) (5%) If the steady state error for a unit-step input is equal to -0.01 and let the compensator be \( C_2(s) = C_1(s) \frac{K_2(s+101)}{s+b} \), what is the ratio of \( \frac{K_2}{b} \)?

(d) (4%) A state space description of the above system \( G(s) \) is the following:
\[
\begin{align*}
\frac{d}{dt} x(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]
where \( A \) and \( C \) are of the observability canonical form. What are \( A \), \( B \), and \( C \)?

(e) (4%) Let \( H(s) = K \) and the compensator = 1 such that the plant \( G(s) \) is compensated by output feedback \( u = -Ky + r \). Find a \( K \) to reach the goal of \( \zeta = 0.5 \) and \( \omega_n = 2 \)? This problem shows you the different compensator design to reach the same goal.

(f) (7%) In part (a), the compensator \( C_1(s) \) is denoted by the following state space description with the parameters \( K_1 \) and \( p \) you find in part (a):
\[
\begin{align*}
\frac{d}{dt} z(t) &= A_z z(t) + B_z e(t) \\
u(t) &= C_z z(t) + D_z e(t)
\end{align*}
\]
Let \( A_z \) and \( C_z \) be of the observability canonical form and the new augmented state be \( [x(t) \ z(t)]^T \). Write down the state space description of the augmented system from \( e(t) \) to \( y(t) \).

(g) (2%) Is the system in part (f) controllable? Give your reason. DO NOT test the controllability by the controllability matrix. It will waste your time.