國立交通大學八十六學年度碩士班入學考試試題

1. (7 %) What's the number of ways to assign 6 identical balls to 4 different boxes with infinite capacity.

2. (8 %) Let the probability of success in a Bernoulli trial be 0.6.
   (a) What is the minimum number of trials that must be performed so that the probability of at least one success is no less than 0.95?
   (b) What is the minimum number of trials that must be performed so that the probability of at least one failure is no less than 0.95?

3. (10 %) Prove that the following well formed formula is a tautology. Do not use truth tables.

$$((P \wedge Q) \Rightarrow R) \wedge ((P \vee Q) \Rightarrow U) \wedge (R \Rightarrow \overline{U})) \quad \Rightarrow \quad (P \Rightarrow \overline{Q})$$

4. (5 %) Prove the following using mathematical induction.

$$\sum_{i=0}^{n} i^3 = n^2(n+1)^2/4$$

5. (10 %) Suppose that

$$T(n) \leq \begin{cases} b & if \ \ 0 \leq n \leq 1 \\ c\,n + \frac{2}{n}\sum_{j=0}^{n-1} T(j) & if \ \ n > 1 \end{cases}$$

   Use mathematical induction to show that $T(n) \leq 2(b+c)n\, log_e n, n \geq 2$.

6. (10 %) A set S is countably infinite iff there exists a one-to-one correspondence between S and the set N of natural numbers. Are the following true or false?
   (a) Every infinite subset of a countably infinite set is countable infinite.
   (b) The union of a infinite number of countably infinite sets is countably infinite.
   (c) {p|p is a prime number} is a countably infinite set.
   (d) No infinite set can be put into one-to-one correspondence with at least one of its proper subsets.
   (e) $\{i + i^2 + i^3 \mid i \in N\}$ is a countably infinite set.

7. Let K_n be the undirected complete graph of n vertices.

   (a) (5%) How many subgraphs does K_5 have?

   (b) (5%) How many simple paths (without repeating vertices) are there between any two vertices of K_8?

   (c) (5%) How many Hamiltonian cycles does K_7 have?

   (d) (5%) What is the minimum number of colors to color edges of K_8 so that adjacent edges are colored differently?

8. (10%) Solve the recurrence relation $t(n) = a\, t(n/b) + c\, n$ for $n \geq 2$ and $t(1) = c$, where you can assume that $n = b^k$.

9. The following is about relations.

   (a) (5%) Find the reflexive transitive closure of the binary relation $R = \{(a,b), (b,c), (b,c), (d,d)\}$ over the set $\{a,b,c,d\}$.

   (b) (5%) What is an equivalence relation?

10. (10%) Show that if $f : X \to Y$ and $g : Y \to Z$ are both one-to-one then $g \bullet f : X \to Z$ is also one-to-one, where $g \bullet f$ is the composition of functions f and g.