1. Let $\vec{F} = x^2yz^2\hat{i} + yz^3\hat{j} + 2xy\hat{k}$.
   
   (a) (5 points) Calculate $\nabla \times \vec{F}$.
   
   (b) (5 points) Calculate $\oint_C \vec{F} \cdot d\vec{l}$ where $C = \{(x-1)^2 + y^2 = R, z = H\}$.

2. Let $A = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

   (a) (5 points) Find the inverse matrix of $A$.
   
   (b) (10 points) Find a matrix $U$ so that $B = U^{-1}AU$ is a diagonal matrix and $|U| = 1$.

3. (10 points) Find the Laurent series for the function $\frac{1}{(z-1)(z-2)}$ in each of the following domains: (1) $|z| < 1$, (2) $1 < |z| < 2$, (3) $|z| > 2$.

4. (15 points) Perform the integral $\int_0^\infty \frac{\sqrt{x}}{x^2 + 1} \, dx$ by contour integral method.

5. Solve the initial-value problem $\frac{dy}{dx} - 2xy = 2$ subjecting to $y(0) = 1$. (10%) 

6. Solve $4y'' + 36y = \csc(3x)$. (10%) 

7. Solve $xy'' + y = 0$. (A Series Solution) (15%) 

8. Put the differential equation $y'' + \lambda y = 0$ (regular Sturm-Liouville problem) in self-adjoint form and derive the orthogonality relation between the eigenfunctions, $y_n(x) = A_n \sin\left(\frac{m\pi}{L} x\right)$, on the interval $(0, L)$. (15%)