1. Suppose that you have just won $6000 and you want to invest it. Two different friends have offered you an opportunity to become a partner in two different ventures, one planned by each friend. Becoming a \textit{full} partner in the first friend’s venture would require $5000 and 400 hours of your time, and your estimated profit would be $4500. The corresponding figures for the second friend’s venture are $4000 and 500 hours, with an estimated profit of $4200. Both friends are flexible in that they will allow you to become any fraction of a full partner that you would like; your share of the profit is proportional to this fraction. You have a maximum of 600 hours available time. Formulate the problem of finding the best combination of investments. (10%)

2. Consider the following problem
\begin{align*}
\text{Max } Z &= 2x_1 + 7x_2 + 4x_3 \\
\text{S.T. } &x_1 + 2x_2 + x_3 \leq 10 \\
&3x_1 + 3x_2 + 2x_3 \leq 10 \\
&x_1, \ldots, x_3 \geq 0
\end{align*}
(a) Construct the dual problem. (5%)
(b) It has been given that $x_2$ and $x_3$ should be the basic variables for the optimal solution of the primal problem. Please derive and identify the complementary basic solution for the dual problem. Then, draw your conclusion about whether these two basic solutions are optimal for their respective problem. (10%)

3. (True or False) Please give a brief explanation no matter what your answer is. (15%, 3% for each)

(a) The best CPF (corner point feasible) solution always is an optimal solution.
(b) The simplex method’s minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.
(c) In a particular iteration of the simplex method, if there is a tie for which variable should be the leaving basic variable, then the next basic feasible solution must have at least one basic variable equal to zero.
(d) If there is no leaving basic variable at some iteration, then the problem has no feasible solution.
(e) When an artificial problem is created by introducing artificial variables and using the Big M method, if all artificial variables in an optimal solution for the artificial problem are equal to zero, then the real problem has no feasible solutions.
4. A company has learned that a competitor is planning to come out with a new kind of product with a great sales potential. This company has been working on a similar product, and research is nearly complete. It now wishes to rush the product out to meet the competition. There are four non-overlapping phases left to be accomplished, including the remaining research that currently is being conducted at a normal pace. However, each phase can instead be conducted at a priority or crash level to expedite completion. The times required (in months) at these levels are as follows:

<table>
<thead>
<tr>
<th>Level</th>
<th>Remaining Research</th>
<th>Development</th>
<th>Design of Manufacturing System</th>
<th>Initial Production and Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Priority</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Crash</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

For these four phases $30,000,000 is available. The cost (in millions) at the different levels is as follows:

<table>
<thead>
<tr>
<th>Level</th>
<th>Remaining Research</th>
<th>Development</th>
<th>Design of Manufacturing System</th>
<th>Initial Production and Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Priority</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Crash</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

Management wishes to determine at which level to conduct each of the four phases to minimize the total time until the product can be marked subject to the budget restriction. Formulate this problem as a shortest path problem and solve it. (10%) 

5. Consider a Two-person Zero-sum game having the following payoff matrix. This game has no saddle point.

<table>
<thead>
<tr>
<th></th>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>1</td>
<td>-3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Player 2</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>Player 2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>
(1). Use the graphical procedure to determine the value of the game and the optimal mixed strategy for Player 1 according to the minimax criterion. (8%)

(2). Determine the optimal mixed strategy for Player 2. (5%)

6. Consider the following Markov chain:

\[
\begin{array}{c|ccc}
\text{state} & 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
2 & 1 & 0 & 0 \\
\end{array}
\]

(1) Is this chain ergodic? Why? (5%)  
(2) Explain why \( \lim_{n \to \infty} p_{11}^{(n)} \) does not exist. (5%)

7. The Hsinchu Police Department has 4 patrol cars. A patrol car breaks down and requires repairing service once every 30 days. The police department has two repair workers, each of whom takes an average of 3 days to repair a car. Breakdown times and repair times are exponential.

(1) Determine the average number of police cars in good condition. (10%)  
(2) Find the fraction of time a particular worker is idle. (5%)

<Note: 精確度到小數第三位。>

8. A company is planning to spend $22,000 on advertising. It costs $4,000 per minute to advertise on television, $1,000 per minute to advertise on radio, and $2,000 per day to advertise on newspapers. If the firm buys x minutes of TV advertising, y minutes of radio advertising, and z days of newspaper advertising, then its revenue in thousands of dollars is given by

\[ f(x, y, z) = -2x^2 - y^2 - z^2 + 8x + 4y + 5z. \]

(1). How can the firm maximize its revenue? And how much is it? (12%)