1. Calculate the minimum kinetic energy of an electron that is localized within a typical nuclear radius of $6 \times 10^{-15}$ m. Use uncertainty principle and assume that the momentum $p$ is at least as large as the uncertainty in $p$. (10 points)

2. A typical diameter of a nucleus is about $10^{-14}$ m. Use the one-dimensional infinite square-well potential to calculate the transition energy from the first excited state to the ground state for a proton confined to the nucleus. Of course, this is only a rough calculation for a proton in a nucleus. (7 points)

3. Consider a particle of kinetic energy $K$ approaching a step function from the left, where the potential barrier steps from 0 to $V_0$ at $x = 0$. Find the penetration distance $\Delta x$, where the probability of the particle penetrating into the barrier drops to $1/e$. (10 points) Calculate the penetration distance for a 5-eV electron approaching a step barrier of 10 eV. (5 points)

4. Figure below shows some of the lowest energy levels of the He atom. They are labeled by their configuration (for example, 1s2p means that the atom has one electron in the 1s level and one in the 2p level). The energy depends somewhat on the orientation of the two electrons’ spins: If the spins are antiparallel, the total spin is zero (quantum number $s_{tot} = 0$); if the spins are parallel, the total spin has $s_{tot} = 1$. For a given configuration, the state with $s_{tot} = 1$ has slightly lower energy.
   (a) Explain why the 1s$^2$ configuration has only $s_{tot} = 0$. (3 points)
   (b) There is a selection rule $\Delta s_{tot} = 0$, that is, transitions in which $s_{tot}$ changes are forbidden. Indicate all allowed transitions on the energy-level diagram. (Don’t forget the selection rule $\Delta l = \pm 1$.) (10 points)
   (c) Which excited levels would you expect to be metastable? (5 points)
5. The vibrational energy levels of the hydrogen molecule (H₂) are well modeled by the harmonic oscillator. Suppose the two hydrogen atoms of the molecule interact with the following potential energy:

\[ V(r) = V_0 \left[ \left( \frac{a}{r} \right)^2 - \frac{a}{r} \right], \]

where \( V_0 \) (units of energy) and \( a \) (units of length) are constants and \( r \) is the separation of the two atoms.

(a) (5 points) Sketch the potential and the ground state wavefunction as functions of the separation, \( r \).

(b) (10 points) Find the equilibrium position, \( r_0 \) in terms of \( a \).

6. The Global Positioning Satellite (GPS) system uses high precision clocks to measure the time-of-flight of radio signals from several different satellites to a receiver on earth.

(a) (10 points) What is the maximum error in the clock time (\( \Delta t \)) to be able to use radio time-of-flight information to locate a receiver to within 1 m?

(b) (10 points) Suppose that in low earth orbit a typical GPS satellite moves with an orbital speed of about 9,000 m/s. at what speed (m/s) will a fixed receiver appear to drift if the clock is not corrected for the relativistic time dilation effect? (Hint: Use the linear approximation for calculation.)

7. Short Answers:

(a) (5 points) Which of the events, \( E_1 \) to \( E_5 \) indicated on the following space-time diagram can send a signal and communicate with event \( E_0 \)?
(b) (5 points) Which pair of functions (if any) in the figure below best represents the nuclear potentials experienced by the protons (P) and neutrons (N) in a heavy nucleus?

![Potential Functions Diagram](image)

(c) (5 points) A non-relativistic particle of mass $m$ and kinetic energy $E > 0$ comes from the left ($x = -\infty$) and hits a potential barrier of height $V_0$ and width $2a$ centered at the origin:

$$V(x) = \begin{cases} 
0 & \text{if } x < -a \text{ (region I)} \\
V_0 & \text{if } -a < x < a \text{ (region II)} \\
0 & \text{if } x > a \text{ (region III)}
\end{cases}$$

Considering just the case of incoming particles from the left with energy less than $V_0$, on the figure below, sketch the potential and a wave function for the scattering case, $0 < E < V_0$, in the region $-3a < x < +3a$. 