1. (10%) Consider the signal \( x(t) = (\cos^2 20\pi t) \cdot (\sin 10\pi t) \).

   (a) (2%) Find its fundamental frequency.
   (b) (6%) Find its complex exponential Fourier series expansion without doing any integration.
   (c) (2%) Find its average power.

2. (10%) The signal

   \[ x(t) = 3 + 4 \cos 10\pi t + 5 \cos 14\pi t + 2 \cos 20\pi t \]

   is sampled at a rate of 30 samples per second. Plot the spectrum of the sampled signal \( x_s(t) \) showing all components for \( |f| < 80 \). Fully explain how \( x(t) \) can be reconstructed from the sampled signal \( x_s(t) \).

3. (15%) A system is described by the input-output relationship

   \[ y(t) = x(t^2) \]

   Is this system:
   (a) (5%) Linear?
   (b) (5%) Causal?
   (c) (5%) Time-invariant?

   Prove your answers.

4. (15%) Use the convolution theorem of Laplace transforms to find \( y(t) = x_1(t) * x_2(t) \), where \( * \) denotes convolution and \( x_1(t) \) and \( x_2(t) \) are given below:

   (a) (7%) \( x_1(t) = e^{-2t} u(t) \) and \( x_2(t) = u(t - 5) \)
   (b) (8%) \( x_1(t) = \cos(5t) u(t) \) and \( x_2(t) = \sin(3t) u(t) \).

   Note that \( u(t) \) is the unit step function.

5. (15%) Obtain the inverse Fourier transform of

   \[ X(f) = \frac{\text{sinc} 2f}{3 + j2\pi f} \]

   where the sinc function is defined as \( \text{sinc} z = \frac{\sin \pi z}{\pi z} \).
6. (15%) The Fourier transforms of two signals, \( x(t) \) and \( y(t) \), are defined as

\[
X(f) = \begin{cases} 
\cos(\pi f), & \text{if } |f| \leq 0.5 \\
0, & \text{otherwise}
\end{cases}
\]

\[
Y(f) = X(f - f_0) + X(f + f_0)
\]

(a) (4%) Find a closed-form expression for \( x(t) \).
(b) (4%) Find a closed-form expression for \( y(t) \).
(c) (7%) Design the system shown in the block diagram in terms of choosing the parameters \( A, f_1, \) and \( f_2 \) so that the output is \( y(t) \).

\[
\cos(2\pi f_2 t)
\]

7. (20%) **Find and plot** the impulse responses of the following systems

(a) (8%) an ideal high-pass filter with transfer function given by

\[
H_{hp}(f) = \left[ 1 - \Pi \left( \frac{f}{2B} \right) \right] e^{-j2\pi t_0 f}
\]

where \( \Pi(\frac{f}{2B}) = \begin{cases} 
1, & |f| \leq B \\
0, & \text{otherwise}
\end{cases} \)

(b) (12%) an ideal notch filter whose phase response is linear,

\[
\theta(f) = -2\pi t_0 f
\]

and amplitude response is shown below

<table>
<thead>
<tr>
<th>( H(f) )</th>
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<tbody>
<tr>
<td>( -f_0 )</td>
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