1. A positive point charge $Q$ is at the center of a spherical dielectric shell of an inner radius $R_i$ and an outer radius $R_o$. The dielectric constant of the shell is $\varepsilon_r$. Find $\vec{E}$, $V$, and $\vec{D}$ as functions of radial distance $R$. (15%)

2. An air coaxial transmission line has a solid inner conductor of radius $a$ and an outer conductor of inner radius $b$ and outer radius $c$. (15%)

   (a) Find $\vec{B}$ as a functions of radial distance $r$.

   (b) Determine the inductance per unit length of the line.

3. The $\vec{H}$ field in air just above a perfect conductor is given by

   $$\vec{H} = 3 \hat{z} + 4 \hat{y} \quad \text{Amp/m}$$

   Find the surface current $\vec{J_s}$ on the surface of the perfect conductor. The conductor occupies the space $x < 0$. (10%)

4. Study the following $\vec{H}$ field in a source-free region:

   $$\vec{H} = \hat{z} H_0 e^{jkz} + j\hat{y} H_0 e^{jkz}.$$ 

   Does it satisfy Maxwell's equations? If so, find the polarization (linear or LHP or RHP) and the corresponding $\vec{E}$ field. If not, explain why not. (10%)

5. For a wave traveling inside a uniform waveguide along $z$-axis with arbitrary cross section on the $x$-$y$ plane, the electrical and magnetic fields can be expressed as

   $$\vec{E}(x, y, z, t) = \text{Re}[\vec{E}^0(x, y) e^{j(\omega t - kz)}], \quad \vec{H}(x, y, z, t) = \text{Re}[\vec{H}^0(x, y) e^{j(\omega t - kz)}].$$

   And of course

   $$\nabla \times (\vec{E}^0(x, y) e^{-jkz}) = -j\omega \mu (\vec{H}^0(x, y) e^{-jkz}), \quad \nabla \times (\vec{H}^0(x, y) e^{-jkz}) = j\omega \varepsilon (\vec{E}^0(x, y) e^{-jkz})$$

   where $\vec{E}^0(x, y)$ and $\vec{H}^0(x, y)$, or $\vec{E}^0$ and $\vec{H}^0$ as a simplification, can be further de-composed into

   $$\vec{E}^0 = \hat{x} E^0_x + \hat{y} E^0_y + \hat{z} E^0_z, \quad \vec{H}^0 = \hat{x} H^0_x + \hat{y} H^0_y + \hat{z} H^0_z.$$ 

   Using the curl equations, please derive step-by-step the four expressions of $E^0_x$, $E^0_y$, $H^0_x$, $H^0_z$ in terms of $E^0_z$, $H^0_z$ like (10%)

   $$H^0_z = -\frac{1}{\omega \mu \varepsilon - k^2_z} \left( \frac{\partial H^0_x}{\partial x} - \frac{\partial E^0_y}{\partial y} \right).$$
6. The standard Ka-band waveguide has the following dimension: \( a = 0.28 \) inch, and \( b = 0.14 \) inch (1 inch = 2.54 cm). The medium inside the waveguide is air. Please find the cutoff frequencies for the first five modes. (10%)

7. (10%)
   
   (a) Please write down the wave number while the wave propagates in a highly conducting media, and briefly describe the behavior of the wave.

   (b) Now if we wrap a food using the aluminum foil and put it into a microwave oven, can we still heat up this food? If yes, please write down the heating mechanism. Here we assume the microwave oven is very robust, and the electrical conductivity of the aluminum foil is \( 3.54 \times 10^7 \) S/m.

8. The resonant frequency of a cubic cavity resonator is 60 GHz when it is excited in the \( \text{TE}_{110} \) mode. Please calculate the dimensions of the resonator. (10%)

9. For an short-circuited 75 \( \Omega \) transmission line of length \( l \), the input impedance at the other end is \( 33j \). Find the length \( l \) in terms of \( \lambda \). (10%)