1. (20 %) For a dynamic plant \( G(s) = \frac{2s + 4}{s^2 + 4s + 20} \),
   
   (a. 6 %) determine its unit-step response \( y_1(t) \) of the open-loop plant simply by applying
   the provided properties as:
   
   \[ L[e^{-at}f(t)] = F(s + a), \quad L[sin \omega t] = \frac{\omega}{s^2 + \omega^2}, \quad L[cos \omega t] = \frac{s}{s^2 + \omega^2} \]
   
   (b. 6 %) with the input \( \sin 2t \), obtain its time response \( y_2(t) \) in a steady state
   \( (t \gg 1 \text{ sec.}) \),
   
   (c. 4 %) to achieve its unit-feedback control system, the gain \( K_1 \) is designed with
   \( \zeta = 0.707 \) and the gain \( K_2 \) is obtained from a tuning process with the unit-step
   response overshoot 4.3%; compare \( K_1 \) with \( K_2 \), which value is larger? why?
   
   (d. 4 %) with a unit-step input, determine the feedback control gain \( K_3 \) to achieve
   \( e_\infty = y(\infty) - y_r = 0 \) with the sensor \( H(s) = 2(1 + 3s) \).

2. (17 %) A motor with its velocity control application is shown as below,

   (a. 3 %) determine the system determinant \( \Delta(s) \)
   
   (b. 3 %) determine the transfer function of \( \frac{\Omega_m(s)}{T_L(s)} \) (expressed with \( \Delta(s) \)),
   
   (c. 3 %) determine the controllers \( K, K_v, \) and \( K_c \) so that a constant torque \( T_L \) results
   in the least effect on \( \omega_m \)
   
   (d. 8 %) obtain its state-space equation with \( u(t) = \begin{bmatrix} \omega_r(t) \\ T_L(t) \end{bmatrix} \) and \( x(t) = \begin{bmatrix} i(t) \\ \omega_m(t) \end{bmatrix} \)
3. (13%) For a plant with a positive gain as 
   \[ KG(s) = \frac{5K}{s(s + 2)(s + 10)}. \]
   (a. 4%) obtain all poles as the unit-feedback system is marginal stable
   (b. 2%) obtain the oscillation period of (a) with a unit-step input,
   (c. 2%) determine the minimum steady-state error of its closed-loop system with a
   unit-ramp input,
   (d. 5%) determine the range of gain \( K \) so that all stable poles are real.

4. (16%) For a unit feedback system with the following open-loop transfer function
   \[ G(s) = \frac{K}{s(s + a)(s + b)}, \] where \( b > 0, a > 0, K > 0. \)
   The following questions are assumed the closed-loop system is stable.
   (a. 5%) Let the input \( r(t) = \cos(\omega t) \). If \( \omega = \sqrt{ab} \) find the steady-state closed-loop
   output.
   (b. 6%) Find the corresponding steady-state closed-loop output when \( \omega \to 0 \) and
   \( \omega \to \infty \), respectively, from the Nyquist Plot.
   (c. 5%) If the phase margin=60\(^\circ\) and the gain-crossover frequency is \( \omega_g \), then find the
   steady-state closed-loop output when the input \( r(t) = \cos(\omega_g t) \).

5. (16%) For a unit feedback system with the open-loop transfer function \( G(s) = \frac{K(s + z)}{s^3} \), where \( z > 0 \),
   (a. 6%) Plot the Root locus for \( 0 < K < \infty \) and \( -\infty < K < 0 \), respectively. Also determine
   the number of RHP pole(s), if exists, for each case.
   (b. 5%) Sketch the Nyquist Plot for \( 0 < K < \infty \). Also determine the number of RHP
   pole(s), if exists.
   (c. 2%) Determine the RHP pole(s), if exists, for \( -\infty < K < 0 \) using the plot in part (b).
   (d. 3%) Draw the Nichols Plot for \( 0 < K < \infty \).
6. (18 %) Consider the following feedback system with the plant \( G_p(s) \) and the controller \( G_c(s) \).

\[
\begin{array}{c}
\text{r} \\
\downarrow \\
+ \\
\downarrow \\
e \\
\downarrow \\
\downarrow \\
G_c(s) \\
\downarrow \\
\downarrow \\
\downarrow \\
G_p(s) \\
\downarrow \\
\downarrow \\
\downarrow \\
y \\
\end{array}
\]

Let
\[
G_p(s) = \frac{1}{(s+2)(s-1)}.
\]

It is desired to design the controller \( G_c(s) \) to meet the following spec.:\( \Theta \) 2% settling time, \( t_s = 4/\zeta \omega_n \), is 2 sec. and \( \Theta \zeta = 1/\sqrt{2} \) for smaller max. overshoot.

(a. 3 %) Determine the dominant closed-loop poles.

(b. 6 %) Choose controller \( G_{c1}(s) = K_1 \frac{s+2}{s+p_1} \). Find \( K_1 \) and \( p_1 \) to meet the above spec.

(c. 3 %) What is the steady-state error for a unit-step input? Explain why if the steady-state error is negative?

(d. 6 %) Since the steady-state error is too big, we find another controller,
\[
G_{c2}(s) = K_2 \frac{(s+2)(s+z_1)}{s+p_2}.
\]

If the new spec. of the steady-state error for a unit-step input, is -1/101, find \( K_2, p_2 \) and \( z_2 \) to meet the spec.

[Hint: 101/4.92=20.5]