1. The following questions are about matrix.

(a) Let \( A + 3AB = 3I \). What is \( A^{-1} \)? (10%)  

(b) Let \( P \) be a symmetric matrix. Is \( (APA^T + (B^TPB)^{-1}) \) a symmetric matrix? (5%)  

(c) What are the eigenvalues of matrix  
\[
\begin{pmatrix}
4 & 1 \\
1 & 2
\end{pmatrix}
\]  

2. The planar coordinates of points A and B are determined by GPS and the results are:

<table>
<thead>
<tr>
<th>Point</th>
<th>X (west-east)</th>
<th>Y (north-south)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100.12 ± 0.01 m</td>
<td>51.32 ± 0.02 m</td>
</tr>
<tr>
<td>B</td>
<td>33.45 ± 0.03 m</td>
<td>67.24 ± 0.01 m</td>
</tr>
</tbody>
</table>

Assume that there is no correlation among all quantities in the above table. Compute the covariance matrix  
\[
\begin{bmatrix}
\sigma_d^2 & \sigma_d\sigma_z \\
\sigma_d\sigma_z & \sigma_z^2
\end{bmatrix}
\]
where \( d \) and \( z \) are distance (A to B) and azimuth (from A to B). (15%)  

3. Two surveyors measured the same angle 51 times each. The standard deviation of measured angles by the first surveyor is 0.81. And, the standard deviation is 1.21 for the second surveyor. Is the first surveyor measured better than the second surveyor at a 0.01 level of significance? (Hint: \( F_{0.01,50,50} = 1.95 \)) (15%)  

4. What is "standard error ellipse"? Please describe its meaning, application, and derive the equations for its computation. (15%)  

5. An angle is observed repeatedly using the same equipment and procedures. Calculate (a) the angle's most probable value (b) the standard deviation of the mean.  
\[105^\circ 20'25", \quad 105^\circ 20'23", \quad 105^\circ 20'18"\]  
\[105^\circ 20'16", \quad 105^\circ 20'27", \quad 105^\circ 20'15"

(15%)
6. A network of differential levels is run from existing benchmark Juniper through new stations A and B to existing benchmarks Red and Rock as shown in the accompanying figure. Calculate the most probable elevations of stations A and B.

(20%)

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Elev. Diff. (m)</th>
<th>σ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juniper</td>
<td>A</td>
<td>26.128</td>
<td>0.009</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>6.873</td>
<td>0.010</td>
</tr>
<tr>
<td>Red</td>
<td>B</td>
<td>11.552</td>
<td>0.005</td>
</tr>
<tr>
<td>Rock</td>
<td>B</td>
<td>-10.852</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Δ (Elev 145.818m)

Rock

Δ (Elev 101.965m)

Juniper

Δ

Red

(Elev 123.409m)