1. A forced undamped oscillator can be expressed by a second-order nonhomogeneous equation of \( y''(t) + 16y = f(t) \). Please find the homogeneous solution (5%). If \( f(t) = e^{4t} \), find the particular solution (5%). If \( f(t) = \sin 4t \), find the particular solution (10%).

2. If temperature is a function of position and time, the rate of change of temperature can be expressed by \( \frac{dT}{dt} = \frac{\partial T}{\partial t} + (\vec{v} \cdot \nabla)T \) where \( \vec{v} \) is the velocity of the flow. If \( T = 10x^2 + 2x^2 + y^2 \) and \( \vec{v} = 2x\hat{i} - y\hat{j} \), please find \( \frac{dT}{dt} \) at the point \((x, y) = (1, 2)\) for time \( t=3 \). (15%)

3. If a function is given by \( f(t) = t + \pi, \quad -\pi < x < \pi \). \( f(t) = f(t + 2\pi) \), please find its Fourier series (15%)

[Note: \( a_0 = \frac{1}{2L} \int_{-L}^{L} f(t) dt, \quad a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} dt, \quad b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} dt \)]

4. Find a general solution of \( xy'' + 3y' + x^{-1}y = 0 \) (15%)

5. Find the moment of inertia \( I \) of a homogeneous spherical lamina

\( S: x^2 + y^2 + z^2 = a^2 \) of mass \( M \) about the \( z \)-axis. (15%)

6. (a) Evaluate \( \int_{0}^{\infty} \frac{\sin x}{1 + x^4} dx \) (10%)

(b) Find the Cauchy principal value of the integral: \( \int_{0}^{\infty} \frac{dx}{x^2 - ix} \) (10%)