1. A forced undamped oscillator can be expressed by a second-order nonhomogeneous equation of $y''(t) + 16y = f(t)$. Please find the homogeneous solution (5%). If $f(t) = e^{4t}$, find the particular solution (5%). If $f(t) = \sin 4t$, find the particular solution (10%).

2. If temperature is a function of position and time, the rate of change of temperature can be expressed by $dT/ dt = \partial T/ \partial t + (\vec{v} \cdot \nabla)T$ where $\vec{v}$ is the velocity of the flow. If $T = 10x^2 + 2(x^2 + y^2)$ and $\vec{v} = 2\alpha x - y\vec{j}$, please find $dT/ dt$ at the point $(x, y) = (1, 2)$ for time $t = 3$. (15%)

3. If a function is given by $f(t) = t + \pi$, $-\pi < x < \pi$, $f(t) = f(t + 2\pi)$, please find its Fourier series (15%) 

[Note: $a_0 = \frac{1}{2L} \int_{-L}^{L} f(t) dt$, $a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} dt$, $b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} dt$]

4. Find a general solution of $xy'' + 3y' + x^{-1}y = 0$ (15%) 

5. Find the moment of inertia $I$ of a homogeneous spherical lamina 

$S: x^2 + y^2 + z^2 = a^2$ of mass $M$ about the $z$-axis. (15%)

6. (a) Evaluate $\int_{-\infty}^{\infty} \frac{\sin x}{1 + x^2} dx$ (10%) 

(b) Find the Cauchy principal value of the integral: $\int_{-\infty}^{\infty} \frac{dx}{x^2 - ix}$ (10%)