1. (a) Let matrix \( A = [a_1, a_2, a_3] \), where \( a_1, a_2, a_3 \) are vectors in \( \mathbb{R}^3 \). If \( 4a_1 - 3a_2 + 2a_3 = 0 \), find \( \det(A) \). (2%)

(b) Given the Adjoint of matrix \( A \) denoted as \( \text{adj}(A) \), find \( \det(A), A, \det(3A^{-1}A^T) \).

Where
\[
\text{adj}(A) = \begin{pmatrix}
2 & 1 & 0 \\
4 & 3 & 2 \\
-2 & -1 & 2
\end{pmatrix}
\]

(6%)

(c) For the linear system \( Ax = b \) (6%)

(i) Find the rank of \( A \) and a basis for the column space of \( A \).

(ii) Find a basis for the null space \( \text{N}(A) \)? What is the dimension of \( \text{N}(A) \)?

Where
\[
A = \begin{pmatrix}
0 & 1 & 1 & 3 & 4 \\
1 & -2 & 1 & 1 & 2 \\
1 & 2 & 5 & 13 & 5 \\
-1 & 3 & 0 & 2 & -2
\end{pmatrix}
\]

2. For this problem, just give the answer, no need to show your computation.

(a) Let \( L \) be the linear transformation of the reflection about the line \( ax + by = 0 \), from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \), where \( a^2 + b^2 \neq 0 \).

Find the matrix representation of \( L \) with respect to the standard basis, find the dimension of the kernel space, and find a basis of the range space. (6%)

(b) Let \( L \) be the linear transformation from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \) having the following matrix representation with respect to the standard basis;

\[
\begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}
\]

Find \( L(L(v)) \), where \( v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \)

Find the dimension of the kernel space, and find a basis of the range space of \( L \). (4%)

(c) Let \( A_1 \) be the matrix representation of a linear transformation \( L \) from \( \mathbb{R}^n \) to \( \mathbb{R}^n \) with respect to the basis \( B_1 \). Let \( B_2 \) be another basis of \( \mathbb{R}^n \), and let \( P \) be the transition matrix corresponding to the change of basis from \( B_1 \) to \( B_2 \).

What is the matrix representation of \( L \) with respect to the basis \( B_2 \)? Express it in terms of \( A_1 \) and \( P \). (2%)
3. Given an m by n matrix $A = [a_{ij}]$, where $a_i \cdot a_j = 0, i \neq j, 1 \leq i, j \leq n$. 
   (a) What can you say regarding the properties of $A^T A$? (at least 3 statements to be made for full score of 4 points) (4%)
   (b) Let $m = n = 3$ and $f(x) = Ax$. Give two specific examples of $A$ and explain the kinds of geometric operations thus involved respectively. (8%)

4. Let $A = [a_{ij}]$ be an nxn matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$. Show that 
   $$\sum_{j=1}^{n} (\lambda_j - a_{jj}) = 0. \quad (6\%)$$

5. Compute $\cos(A)$ for $A = \begin{bmatrix} -2 & -6 \\ 1 & 3 \end{bmatrix}$. \quad (6\%)

6. (5%) A box contains $w$ white balls and $b$ black balls. Balls are drawn randomly from the box without replacement until a black ball is drawn. If $n < w$, then what is the probability that exactly $n$ balls are drawn?

7. (10%) Given that $P(X=a) = r$, $P(\max(X,Y)=a) = s$, and $P(\min(X,Y)=a) = t$, show that you can determine $u = P(Y=a)$ in terms of $r$, $s$, and $t$.

8. (8%) Let $X$ be the number of random numbers selected from $\{0, 1, 2, \ldots, 9\}$ independently until 0 is chosen. Find the probability mass functions of $X$ and $Y = 2X + 1$.

9. (20%) Alice and John are having a date tonight. John arrives at random from 7:00pm to 8:00pm. Let $X$ be the arrival time of John. Alice arrives at random from time $X$ to 8:00pm. Let $Y$ be the arrival time of Alice.
   (a) If Alice does not show up within 15 minutes after John arrived, John leaves without a date. What is the probability that Alice and John have a date tonight?
   (b) What is the expected waiting time (either meeting with Alice or leaving without a date) of John?

10. (7%) N fair six-sided die are tossed independently. Let $X_i$ be the point of the $i$th dice. Use the Chebyshev inequality to estimate the probability that the average point of the dice is in between 3 and 4.