1. (a) Find a particular real solution to $\dot{x} + 5x = 4e^{-t} \cos(2t)$ (5%)
(b) Find the general real solution to $\ddot{x} + 2\dot{x} + 2x = 2t^2 + 2$ (5%)
(c) Find the solution to $\ddot{x} + 4\dot{x} + 4x = 4$ such that $x(0) = 0$ and $\dot{x}(0) = 0$ (5%)
(d) What is the solution of the O.D.E. $\frac{dy}{dx} = 2xy^2$ for which $y(0) = 1$ (10%)

2. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 20e^{2x+y}$ (15%)
(b) Solve P.D.E. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin x\pi \omega$ for $0 \leq x \leq 1$, $t \geq 0$ (10%)
\[
 u(0, t) = 10 \quad \frac{\partial u}{\partial x}(1, t) = 0 \quad u(x, 0) = 110
\]

3. (15%) Solve the following system of linear equations using (a) inverse matrix method (i.e., $Ax = b \Rightarrow x = A^{-1} b$), and (b) Cramer’s Rule.
\[
\begin{align*}
3x_1 + x_2 + x_3 + 2x_4 &= 1 \\
x_1 + 3x_2 + 2x_3 + 3x_4 &= 4 \\
2x_1 + x_2 + 3x_3 + 3x_4 &= 3 \\
x_1 + 2x_2 + x_3 + 2x_4 &= 2
\end{align*}
\]

4. Find the PLDU factorization of a matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 6 & 3 \end{bmatrix}$, and use the PLDU factors of $A$ to solve
\[
Ax = \begin{bmatrix} 6 \\ 8 \\ 3 \end{bmatrix}, \quad (15%)
\]

5. Find the sixth roots of $-1 + j\sqrt{3}$ (i.e., $(-1 + j\sqrt{3})^{1/6}$), and show that three of the roots are the solution of $\sqrt{2}z^2 + 1 + j\sqrt{3} = 0$. (20%)