1. (15 %) Use Green's theorem to find the line integral
\[ \oint_C y \, dx + x^2 \, y \, dy, \]
where \( C \) is the closed curve formed by \( x^2 = 4x \) and \( y^2 = 2x \) between (0,0) and (2,2).

2. (15 %) Find the extrema of the function
\[ f(x) = \sin x + \cos x. \]
Sketch the graph for \( 0 \leq x \leq 2\pi \).

3. (15 %) A cylindrical tank of 4 m in diameter and 10 m long is half full of water. If the tank is lying on its side, find the total vertical force \( F \) (in Newton) exerted by the water on the tank. (density of water \( \approx 1000 \) kg/m\(^3\))

![Diagram of a cylindrical tank with forces](attachment:image.png)

4. (15 %) Evaluate the inverse Laplace transform, \( g(t) \), of the following function:
\[ \frac{e^{-4s}}{s + 2} \]
Plot \( g(t) \).

5. (20 %) (i) Let \( A \) be a \( n \times n \) matrix. If for some nonzero \( n \times 1 \) matrix \( X \), \( AX = \lambda X \), then what is \( X \) called? \( \lambda \) is a real or complex number.
(ii) What is the characteristic polynomial of \( A \)?
(iii) What is the name of the solutions to the characteristic polynomial?
(iv) Find all the eigenvalues and the corresponding eigenvector of each eigenvalue for the following matrix:
\[
\begin{bmatrix}
-5 & 0 \\
1 & 2
\end{bmatrix}
\]
6. (20%) (i) Derive the trapezoidal rule for approximating the integral as

\[ \int_a^b f(x)dx = \frac{b-a}{n}(y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n) \]

Where \( y_j = f(x_j) \) for the chosen partition \( a = x_0 < x_1 < x_2 < ... < x_{n-1} < x_n = b \), and \( h \) is the spacing between successive partition points. \( h = \frac{b-a}{n} \), and \( n \) is an integer.

(ii) Approximate the following integral by using the trapezoidal rule with \( n = 2 \) and 4:

\[ \int_0^1 \frac{dx}{\sqrt{x^2 + 2x + 12}} \]