1. Evaluate the following indefinite integrals
(a) \[ \int x^2 e^{\alpha x} \, dx \] (10%)
(b) \[ \int \frac{1}{\sin x} \, dx \] (10%)

2. Show that
(a) If \( B \) is a symmetric matrix, show that matrix \( (A^T BA - ABA^T) \) is a symmetric matrix (5%)
(b) If matrix \( A \) satisfies \( A = AA^T \), then the eigenvalues of \( A \) are 0 or 1. (5%)

3. Solve the following differential equations
(a) \( y'' + y = x \) (10%)
(b) \( y'' - 2y' + y = e^x + x \) (10%)

4. Solve the following partial differential equation for \( u(x, y) \):
\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u \] (15%)

5. Show that
\[ \frac{1}{\sqrt{1 - 2xu + u^2}} = \sum_{n=0}^{\infty} P_n(x)u^n \]
This function on the left is called a generating function of the Legendre polynomials.

Hint: Start from the binomial expansion of \( \frac{1}{\sqrt{1 - v}} \), set \( v = 2xu - u^2 \), multiply the power of \( 2xu - u^2 \) out, collect all the terms involving \( u^n \), and verify that the sum of these terms is \( P_n(x)u^n \). (15%)

6. In each case show that the given set is orthogonal on the given interval \( I \) and determine the corresponding orthonormal set.
(a) \( 1, \cos x, \cos 2x, \cos 3x, \ldots \), \( I: 0 \leq x \leq 2\pi \) (10%)
(b) \( \sin \frac{n\pi}{L} x \) \( (n=1, 2, 3, \ldots) \), \( I: -L \leq x \leq L \) (10%)