1. (15%) Prove the convolution theorem of the Laplace transform.

2. (20%) Find the general solution for
\[ x^3 y''' - 2x^2 y'' + 3xy' - 3y = 2x + 3x^3, \]
where \( y \) is a function of \( x \).

3. (15%) Find the general solution for
\[ y' = (y + x)(y + x - 2) - 1 \]
where \( y \) is a function of \( x \).

4. (10%) Given a function \( \Phi(x, y) = k \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right) \), find the directional derivative of \( \Phi \) along its boundary curve \( C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).

5. (10%) Given a vector field \( \mathbf{F} = xi + yj + zk \), evaluate the surface integral
\[ \iint_S \mathbf{F} \cdot \mathbf{n} \, dA \]
over the surface \( S: r = (u \cos v, u \sin v, uv) \) \( 0 \leq u \leq 2, -\pi \leq v \leq \pi \) where \( \mathbf{n} \) is the outer unit vector of \( S \).
(Note: \( \iint_S \mathbf{F} \cdot \mathbf{n} \, dA = \iiint_r \mathbf{F}(r(u, v)) \cdot \mathbf{N}(u, v) \, du \, dv \) where \( \mathbf{N} = \mathbf{r}_u \times \mathbf{r}_v \))

6. Given \( A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \)

(a) Find the eigenvalues and eigenvectors of \( A \). (10%)

(b) Let \( \mathbf{P} \) be the eigenvector matrix consisting of the eigenvectors of \( A \), find \( \mathbf{P}^{-1} \) using the method of Gauss-Jordan elimination. (10%)

(c) Show that the eigenvalues of the similarity matrix \( \hat{A} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P} \) are the same as \( A \) and the eigenvectors of \( \hat{A} \) are \( \mathbf{P}^{-1} \mathbf{x} \) where \( \mathbf{x} \) is the eigenvector of \( A \). (10%)