1. (15%) Prove the convolution theorem of the Laplace transform.

2. (20%) Find the general solution for

\[ x^3y''' - 2x^2y'' + 3xy' - 3y = 2x + 2x^3, \]

where \( y \) is a function of \( x \).

3. (15%) Find the general solution for

\[ y' = (y + x)(y + x - 2) - 1 \]

where \( y \) is a function of \( x \).

4. (10%) Given a function \( \Phi(x, y) = k\left(\frac{x^2}{a^2} + \frac{y}{b^2} - 1\right) \), find the directional derivative of \( \Phi \) along its boundary curve \( C : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).

5. (10%) Given a vector field \( \mathbf{F} = xi + yj + zk \), evaluate the surface integral

\[ \iint_S \mathbf{F} \cdot n \, dA \]

over the surface \( S : \mathbf{r} = [u \cos v, u \sin v, u^2] \) where \( 0 \leq u \leq 2, -\pi \leq v \leq \pi \) and \( n \) is the outer unit vector of \( S \).

(Note: \( \iint_S \mathbf{F} \cdot n \, dA = \iint_S \mathbf{F}(r(u, v)) \cdot N(u, v) \, du \, dv \) where \( N = r_u \times r_v \).)

6. Given

\[
\begin{bmatrix}
-2 & 2 & -3 \\
2 & 1 & -6 \\
1 & 2 & 0
\end{bmatrix}
\]

(a) Find the eigenvalues and eigenvectors of \( \mathbf{A} \). (10%)

(b) Let \( \mathbf{P} \) be the eigen-matrix consisting of the eigenvectors of \( \mathbf{A} \), find \( \mathbf{P}^{-1} \) using the method of Gauss-Jordan elimination. (10%)

(c) Show that the eigenvalues of the similarity matrix \( \hat{\mathbf{A}} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P} \) is the same as \( \mathbf{A} \) and the eigenvectors of \( \hat{\mathbf{A}} \) is \( \mathbf{P}^{-1} \mathbf{x} \) where \( \mathbf{x} \) is the eigenvector of \( \mathbf{A} \). (10%)