1. (10%) A magnetic-ball suspension system is shown below. If the levitation force is \( f(t) = K \frac{i(t)}{y(t)^2} \) and the ball is operated around \( y_0 \), obtain the Laplace transfer function \( \frac{\Delta Y(s)}{\Delta I(s)} \) of its linearized model.

![Diagram of magnetic-ball suspension system]

2. For a system as \( G(s) = \frac{4}{s(s+2)^2} \)

(a) (5%) obtain its unit-step response \( y(t) \);

(b) (5%) for the unit-feedback controller \( \frac{K}{s^r} \), list all conditions of the \( K \) and \( u \) that the steady-state error due to the input \( r(t) = 2t^2 \) is less than 0.001.

![Diagram of unit-feedback control system]

(c) (5%) with the non-unit feedback \( H(s) = 0.8(1+s) \), determine the range of the gain controller \( K_p \) so that its steady-state error \( e_s = r - y(\infty) \) with a unit-step input is less than 0.01.

![Diagram of non-unit feedback control system]
3. For a plant as \[ G(s) = \frac{2(s-1)}{(s+2)^2(s^2 + 2s + 2)} \]

(a) (3%) as shown below, determine the range of the unit-feedback gain \( K \) to achieve a stable system,

(b) (4%) determine the range of the gain \( K \) that only one unstable pole exists in the system,

(c) (4%) determine all the root-locus crossing points on the \( j\omega \)-axis,

(d) (4%) determine the departure angle of its root locus at the complex poles,

\[ \frac{K}{s+a} \]

(c) (10%) design the controller \( \frac{K}{s+a} \) as shown below that a pair of the system poles are at \(-1 \pm 1.5j\)

4. (15%) Consider the feedback system below.

(a) (4%) Write the state equation for the closed-loop system using \( r \) as input, \( y \) as output, and \( x = [x_1 \ x_2]^T \) as state variable.

(b) (3%) For what values of \( K_1 \) and \( K_2 \) is the closed-loop system stable?

(c) (4%) Suppose \( K_1 \) and \( K_2 \) are such that the closed-loop system is stable and has an underdamped response. If we keep \( K_1 \) constant, and slightly increase \( K_2 \), how would the rise time, overshoot, steady state error, settling time of the closed-loop unit-step response (from \( r \) to \( y \), change as \( K_2 \) is increased.

(d) (4%) Repeat (c) except now \( K_2 \) is kept constant and \( K_1 \) is increased.
5. (20%) Consider the feedback system shown below, where the time delay is 0.12 seconds.

(a) (5%) Suppose \( K(s) = 1 \). Is the system stable? What is the phase margin of the feedback system?

(b) (5%) Suppose you are asked to design a first-order controller \( K(s) \) so that the following specifications are met:
   - steady state error of unit-step response \( \leq 10\% \)
   - phase margin \( \geq 40^\circ \)
   - gain crossover frequency \( \geq 10 \text{ rad/sec} \)

Among the controller types, PI, lead, lag, lead-lag, which would you choose for your design and why?

(c) (10%) Design a first-order controller of the form
   \[
   K(s) = k \frac{s + z}{s + p}
   \]

so that the specifications in (b) are met.

6. (15%) Are the following statements true (T) or false (F)? Every correct answer gets 3 points and every wrong answer takes 1 point away from your score (until there are no more points left.)

(a) (3%) If the step response of a unity-feedback system has zero steady state error, then there must be at least one integrator in the loop.

(b) (3%) If a unity-feedback system tracks unit-ramp with no steady state error, then its unit-step response must have positive overshoot.

(c) (3%) A phase-lag controller can be used to increase the phase margin of a feedback system.

(d) (3%) If a feedback system is stable, then its loop transfer function must also be stable.

(e) (3%) In controller design, open-loop unstable pole-zero cancellations are not allowed mainly because exact cancellations are not possible.