1. (16%)  
The Laplacian of function $u(x, y, z)$ in rectangular coordinates is given by  
\[ \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}. \]  
(a) The relationship between the cylindrical coordinates of a point in space and its rectangular coordinates is given by $x = r \cos \theta$, $y = r \sin \theta$, and $z = z$. Please derive the Laplacian of $u$ in cylindrical coordinates. (4%)  
(b) In the spherical coordinates, $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, and $z = r \cos \theta$. Please derive the Laplacian of $u$ in spherical coordinates. (4%)  
(c) Please find the electrostatic potential $u$ in the circular cylinder shown in Fig. 1c. (4%)  
(d) Please find the electrostatic potential $u(r, \theta)$ in the sphere shown in Fig. 1d. (4%)

[Figures 1c and 1d]

2. (8%)  
(a) A finite line charge of length $2L$ carrying uniform line charge density $\rho_l$ is coincident with the $x$-axis. Please (a1) determine $V$ in the plane bisection the line charge and (a2) $E$ from $\rho_l$ directly by applying Coulomb's law. (4%)  
(b) A parallel-plate capacitor of width $w$, length $2L$, and separation $d/2$ has a solid dielectric slab of permittivity $\varepsilon$ in the space between the plates. The capacitor is charged to a voltage $V_0$ by a battery, as shown in Fig. 2. Assuming that the dielectric slab is withdraw to the position, determine the force acting on the slab (b1) with the switch closed and (b2) after the switch is first opened. (4%)

[Figure 2]
3. (18%, 3% for each problem)

For each of the following electric fields, please find the charge distribution which produces the field, using Gauss' law in differential form:

\[
\begin{align*}
\mathbf{E} &= \begin{cases} 
- \frac{\rho_{10}}{\varepsilon_0} \mathbf{i}_r, & -\infty < z < 0 \\
- \frac{\rho_{10}}{3\varepsilon_0} \mathbf{i}_r, & 0 < z < a \\
\frac{\rho_{10}}{\varepsilon_0} \mathbf{i}_r, & a < z < \infty
\end{cases} \quad \text{cartesian coordinates}
\end{align*}
\]

\[
\mathbf{E} = \begin{cases} 
\frac{1-e^{-r/a}}{\varepsilon_0 r} \mathbf{i}_r, & 0 < r < \infty
\end{cases} \quad \text{cylindrical coordinates}
\]

\[
\mathbf{E} = \begin{cases} 
0, & 0 < r < a \\
\frac{Q}{4\pi \varepsilon_0 r^2} \mathbf{i}_r, & a < r < b \\
0, & b < r < \infty
\end{cases} \quad \text{spherical coordinates}
\]

For the following surface charge distribution, please obtain the potentials:

\[
\rho_s = \begin{cases} 
\rho_{10}, & z = a \\
-\rho_{10}, & z = -a
\end{cases} \quad \text{cartesian coordinates}
\]

\[
\rho_s = \begin{cases} 
\rho_{10}, & r = a \\
-\frac{\rho_{10}}{a}, & r = b
\end{cases} \quad \text{cylindrical coordinates}
\]

\[
\rho_s = \begin{cases} 
\rho_{10}, & r = a \\
-\rho_{10} \frac{a^2}{b^2}, & r = b
\end{cases} \quad \text{spherical coordinates}
\]

where $\rho_{10}$ is a constant.
4. (10%)
(a) A circular loop of wire of radius \( a \) lying in the \( xy \) plane with its center at the origin carries a current \( I \) in the \( \phi \) direction. Find magnetic filed \( B \) at the point \( (0, 0, z) \). Verify yours answer by letting \( z \to 0 \). (5%)
(b) Two circular loops of filamentary wire each of radius \( a \) and with their centers on the \( z \) axis are situated parallel to and symmetrically about the \( xy \) plane with the separation equal to \( 2b \) as shown in Fig. 4b. The loops carry a current of \( I \) amp each in the \( \phi \) direction. (i) Obtain the expression for magnetic filed \( B \) at a point on the \( z \) axis. (ii) Show that if \( b = a/2 \), the first three derivatives of \( B \) evaluated at the origin are equal to zero. (5%)

![Figure 4b.](image)

5. (15%) Please state and prove the uniqueness theorem of Poisson boundary-value problem.

6. (8%)
Two conducting spheres of radii \( r_1 \) and \( r_2 \) that have a very high conductivity are immersed in a poorly conducting medium (for example, they are buried very deep in the ground) of conductivity \( \sigma \) and permittivity \( \varepsilon \). The distance, \( l \), between the spheres is very large in comparison with the radii. Determine the resistance between the conducting spheres.

7. (13%)
(a) (6%) An air coaxial transmission line has a solid inner conductor of radius \( r_1 \) and a very thin outer conductor of inner radius \( r_2 \). Please find the inductance per unit length of the line.
(b) (7%) Starting from the input impedance of an open-circuited lossy transmission line, please find the expressions for the half-power bandwidth and the \( Q \) of a low-loss line with \( l = n \lambda /2 \).
8. (12%, 3% for each problem)
   (a) What are the diamagnetic, paramagnetic, and ferromagnetic materials?
   (b) Express the transformer emf induced in a stationary loop in terms of 
       time-varying vector potential $\mathbf{A}$. 
   (c) Write the boundary conditions that exist at the interface of free space and a 
       magnetic material of infinite (an approximation) permeability. 
   (d) For the propagating waves in a uniform waveguide, what are the transverse 
       electromagnetic waves, transverse magnetic waves, and transverse electric waves? 
       Please explain them according to whether $E_z$ or $H_z$ exists.