1. Let $F(x)$ be the value of the cumulative distribution function of the continuous variable $X$ at $x$.
   a. Find the probability density function of $Y = F(X)$. (10%)
   b. Describe one possible application of the result in a. (10%)

2. If $X$ is a random variable having a normal distribution with mean $\mu$ and the variance $\sigma^2$. Derive the following moments for $X$:
   a. $E\{X\}$ (10%)
   b. $E\{(X - \mu)^4\}$ (10%)

3. Assume $X$ and $Y$ are independent random variables with $X \sim N(0, 1)$ and $Y \sim \text{Bernoulli}(p)$, where $0 < p < 1$. Define $Z = X$ if $Y = 1$ and $Z = -X$ if $Y = 0$.
   a. Find the probability function of $Z$. (10%)
   b. Find the covariance of $X$ and $Z$. (10%)

4. Suppose that $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ are independent random samples from normal distributions with respective unknown means $\mu_X$, $\mu_Y$ and common variance $\sigma^2$.
   a. Find the maximum likelihood estimators of $\mu_X$, $\mu_Y$ and $\sigma^2$. (10%)
   b. Construct a likelihood ratio test of $H_0$: $\sigma^2 = \sigma_0^2$ against $H_1$: $\sigma^2 = \sigma_1^2$. (10%)

5. Consider a distribution having a probability mass function of the form
   
   $p(x, p) = p^x(1-p)^{1-x}, x = 0, 1.$

   Let $H_0$: $p = 0.05$ and $H_1$: $p > 0.05$.
   a. Find the uniformly most powerful test of $H_0$ against $H_1$. (10%)
   b. Use the central limit theorem to determine the sample size $n$ of a random sample so that a uniformly most powerful test of $H_0$ against $H_1$ has a power function $g(p)$, with approximately $g(0.05) = 0.05$ and $g(0.10) = 0.95$. (10%)