1. Please use Gauss elimination to show why the system with the following three equations has no solution. (10 分)
   \[
   \begin{align*}
   3x + 2y + z &= 3 \\
   2x + y + z &= 0 \\
   6x + 2y + 4z &= 6
   \end{align*}
   \]

2. A particle is moving in a tube. Assume that the position of the particle \( x \) (m) at time \( t \) (sec) can be approximately expressed by \( x = c_1 + c_2 \ t \). Suppose that measurements of the position \( x \) (unit: m) at time \( t \) (sec) are
   \[
   \begin{array}{cccccc}
   t & 0 & 3 & 5 & 8 & 10 \\
   x & 0 & 30 & 40 & 70 & 80 \\
   \end{array}
   \]
   (1) Please apply the method of least squares to estimate the speed, \( c_2 \).
   (2) If \( c_1 \) is removed from the equation, what is your new answer for the estimated speed?
   (20 分)

3. Please use vectors to prove that the line which joins one vertex of a parallelogram to the midpoint of an opposite side divides the diagonal in the ratio 1:2. (20 分)
4. The general nth order differential equation is
\[ F(x, y, y', \ldots, y^{(n)}) = 0 \]
(i) What is the form of the above equation when it is linear?
(ii) How to determine if the linear nth order differential equation is homogeneous?
(iii) If functions \( y_1(x), \ldots, y_n(x) \) are solutions of the homogeneous linear nth order differential equation, when can we call these functions linearly dependent or linearly independent over an interval \( J \)?
(iv) Following the above problem, what is the general solution of the homogeneous linear nth order differential equation if the solutions \( y_1(x), \ldots, y_n(x) \) are linearly independent?

(13 分)

5. \( \phi = e^y \cos(yz) \) is a scalar field. (i) Find a unit vector from the point \((1, 1, \pi)\) in which \( \phi \) has its maximum rate of change, and also find the magnitude of this change.
(ii) What is the tangential plane through the point \((1, 1, \pi)\).

(13 分)

6. Evaluate the inverse Laplace transform \( \mathcal{L}^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right] \) by use of the convolution theorem. (12 分)

7. (i) Determine the transformation from cylindrical \((r, \theta, z)\) to 3-D rectangular \((x, y, z)\) coordinates. (ii) Prove that a cylindrical coordinate system is orthogonal.

(12 分)