Physical constant you might need: Planck's constant $h = 6.63 \times 10^{-34}$ Joule sec.

選擇題(每題 5 分，共計 40 分；每答錯 1 題，則倒扣 1 分)

1. At what speed will the relativistic kinetic energy of a particle be equal to its rest mass energy?
   (a) 0.560c
   (b) 0.750c
   (c) 0.866c
   (d) c
   (e) 1.150c

2. 10000 muons approach the earth surface at speed $\sqrt{8/9} c$ from an altitude $L = 849$ m measured by an observer fixed on the earth. If the half-life of muon is $10^6$ s. How many muons arrive at the earth surface?
   (a) 1250
   (b) 2500
   (c) 5000
   (d) 7500
   (e) 10000

3. What is the $(n, \ell, m_\ell)$ configuration for the first excited electronic state of helium (2 protons, 2 neutrons, 2 electrons)
   (a) (1, 0, 0) (1, 0, 0)
   (b) (1, 0, 0) (1, 1, 0)
   (c) (1, 0, 0) (1, 0, 1)
   (d) (1, 0, 0) (2, 0, 0)
   (e) (2, 0, 0) (2, 0, 0)

4. An electron, proton, neutron and an $\alpha$-particle all have energy of the same amplitude. Which of the following lists is in order of increasing de Broglie wavelength, shortest to longest?
   (a) All have wavelengths of the same magnitude
   (b) $\alpha$-particle, neutron, proton, and then the electron
   (c) $\alpha$-particle, proton, neutron, and then the electron
   (d) electron, neutron, proton, and then the $\alpha$-particle
   (e) electron, proton, neutron, and then the $\alpha$-particle

5. The number of electronic states in the He$^+$ ion corresponding to the principle quantum number $n = 5$ is
   (a) 9
   (b) 11
   (c) 18
   (d) 25
   (e) 50
6. Ultraviolet light of wavelength 350 nm and intensity 1.0 W/m² is shined on an unknown metal. The maximum kinetic energy of the photoelectron is 1.3 eV. What kind of metal is it? (Note that the work function for K, Ca, Cu, Pd and Pt are 2.3, 3.2, 4.7, 5.4, and 6.4 eV, respectively)
   (a) Potassium (K)
   (b) Calcium (Ca)
   (c) Copper (Cu)
   (d) Pladium (Pd)
   (e) Platinum (Pt)

7. The high energy electron beam can be collimated at the semiconductor which has clean surface of highly ordered atoms. The electron beam is diffracted by the ordered atom and forms streaky pattern at a distance of several tens centi-meter away. Suppose the spacing between atoms is about 0.05 nano-meter. What is the voltage that should be applied to accelerate the electron in order to observe the diffraction pattern?
   (a) 10⁻⁶ V
   (b) 10⁻² V
   (c) 1 V
   (d) 10³ V
   (e) 10⁸ V

8. Which of the following functions cannot be the solution of Schrödinger’s equation for all values of x?
   (a) sin(x)
   (b) tan(x)
   (c) e⁻⁻²
   (d) xe⁻⁻²
   (e) x²e⁻⁻²

B. 計算題(每題15分，共計60分)
9. (a) Write down the Schrödinger’s equation for the one dimensional simple harmonic motion of a particle whose mass is m and the force constant is k. (5points)
   (b) Suppose Ce⁻⁻² is a wave-function of the Schrödinger equation. Express α in terms of m, ω, and h. (Note that: ω=√k/m) (5points)
   (c) Find the eigen-energy of this wave-function. (5points)
10. The wave function for a particle of mass $m$ moving in a certain potential $V(x)$ is given by:

$$\psi(x, t) = \begin{cases} \text{Ax} e^{-\alpha x} e^{-i C \nu x} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

where $A$, $B$, and $C$ are real constants. $\psi(x, t)$ is a properly normalized wave function that obeys the time-dependent Schrödinger equation for the potential $V(x)$.

(a) Is the particle bound or free? (3 points)

(b) Find the potential $V(x)$ and the constant $C$ in terms of $A$, $B$, $m$, and $\hbar$. Make a sketch of $V(x)$. (12 points)

11. Use the Bohr atomic model to derive the orbital radii in Bohr atom. (15 points)

12. Magnetic resonance imaging (MRI) depends on the absorption of electromagnetic radiation by the nuclear spin of hydrogen atoms in our bodies. The nucleus is a proton with spin $1/2$ and magnetic moment $\mu_B = 1.41 \times 10^{-26}$ Joule/Tesla.

(a) The person to be scanned by an MRI machine is placed in a strong magnetic field with $B = 1.5$ Tesla. What is the energy difference between two spin states in this field? (8 points)

(b) What is wavelength $\lambda$ of photons that can be absorbed by this energy difference? Which kind of EM wave is it? (X-ray, UV, visible light, Radio) (7 points)