Please note:

- Order your answers according to problem numbers. Answers should be in sequential order.
- Your answers may be in either English or Chinese. Use either English or Chinese where appropriate.
- If you think that the conditions given in a problem are incomplete, then make proper assumptions and state clearly your assumptions and reasons. If you assume certain conditions, state clearly which assumptions you have made.

Problems:

1. (Total=6%) Find the Hilbert transform of the following signals. Show process of computation clearly.
   (a) (3%) \( x_1(t) = \delta(t) \) (the unit impulse function).
   (b) (3%) \( x_2(t) = \sin(2\pi t) \sin(20\pi t) \).

2. (Total=9%) The frequency response of a continuous-time LTI system is
   \[
   H(f) = \frac{j4\pi f + 3}{(j2\pi f + 1)(j2\pi f + 2)}.
   \]
   Determine
   (a) (3%) its impulse response (with clear derivation),
   (b) (3%) its causality (with explanation), and
   (c) (3%) its differential-equation description (with clear derivation).

3. (Total=10%) Consider a discrete-time system described by the block diagram below, where “D” denotes the unit delay.

   ![Block Diagram]

   (a) (4%) Find the transfer function \( H(z) = \frac{Y(z)}{X(z)} \). (Show derivation clearly.)
   (b) (3%) Is the system stable and why?
   (c) (3%) Given \( x[n] = -\infty, \ldots, \infty \), if we want to compute the output \( y[n] \) starting from \( n = 0 \), what is the minimum initial condition of this system we need to know?

4. (Total=10%) A first-order PLL contains three building blocks: phase detector, loop filter, and VCO, with the input and output signals for each building block given as:
   \[
   r(t) \cos(\omega_0 t + \theta(t)) \rightarrow \text{Phase Detector} \rightarrow e(t),
   e(t) \rightarrow \text{Loop Filter } h(t), \quad H(f) \rightarrow y(t) = \frac{1}{K_0} \frac{d}{dt}[\theta'(t)],
   y(t) \rightarrow \text{VCO with Gain } K_0 \rightarrow x(t) = -2 \sin[\omega_0 t + \theta'(t)],
   \]
   where note that \( \theta'(t) \) is not necessarily the first-order derivative of \( \theta(t) \).
   (a) (5%) What is the loop bandwidth of the first-order PLL?
   (b) (5%) What is the loop equation?

5. (5%) What will be the bandwidth requirements for noncoherent FSK and for DPSK with data rate \( R = 1 \) Gbps? Use null-to-null RF bandwidths. Assume that the FSK “tones” are separated by \( 2/T \) Hz, where \( T \) is the symbol period. The modulation schemes are binary.
6. (5%) Give the schematic diagram of an early-late gate synchronizer and briefly describe its operation.

7. (5%) Consider the sequence 110 111 000 101 010. It is differentially encoded to 1 110 000 100 110 and assume that the differentially encoded sequence is used to biphase modulate a sinusoidal carrier of arbitrary phase. Give the schematic diagram of a demodulator of the DPSK.

8. (Total=16%) Consider the system as shown below.

\[ x_r(t) = s(t) + n(t) \quad \longrightarrow \quad \text{Predetection (IF) Filter} \quad \longrightarrow \quad \text{Postdetection Lowpass Filter} \quad \longrightarrow \quad y(t) \]

\[ \cos(2\pi \times 10^5 t) \]

The frequency responses of the predetection filter and the postdetection filter are

\[ H_{\text{pre}}(f) = \begin{cases} 
1, & 97500 \leq |f| \leq 102500 \text{ Hz}, \\
0, & \text{otherwise}, 
\end{cases} \]

and

\[ H_{\text{post}}(f) = \begin{cases} 
1, & 0 \leq |f| \leq 2000 \text{ Hz}, \\
0, & \text{otherwise}, 
\end{cases} \]

respectively. Assume \( n(t) \) is additive white Gaussian noise with double-sided power spectral density \( \frac{1}{2}N_0 = 0.5 \times 10^{-12} \) W/Hz, while the spectrum of \( s(t) \) is as plotted below:

(a) (4%) Find the autocorrelation function of the noise at the output of the predetection filter.

(b) (4%) Find the autocorrelation function of the noise at the output of the postdetection filter.

(c) (4%) Find the signal power at the output of the postdetection filter.

(d) (4%) Find the noise power at the output of the postdetection filter.

9. (Total=9%) Assume the signal \( m(t) = \text{sinc}(10t) + \text{sinc}^2(10t) \) is to be transmitted from point A to point B.

(a) (4%) If DSB is employed and the carrier \( A \cos(2\pi f_0 t) \) has a power content of 100 W, what are the power and bandwidth of the modulated signal?

(b) (5%) If FM is employed and the modulated signal is expressed as \( x(t) = 20 \cos(20000\pi t + 50\pi \int_{-\infty}^{t} m(\alpha) d\alpha) \), what are the power and the approximate bandwidth of the modulated signal?

10. (6%) Consider baseband digital transmission using PAM with symbol period equal to \( T \). Let the combined frequency response of the transmitter filter, the channel, and the receiver filter be as shown below:

\[ \begin{array}{c}
\text{2} \\
\text{-W} \\
\text{W} \\
\text{f (Hz)}
\end{array} \]

where \( W = 1.5/T \). Does this satisfy Nyquist's first criterion for zero ISI? Why?
11. (6%) Compare zero-forcing and MMSE (minimum mean-square error) equalization in terms of their objectives.

12. (7%) Consider transmission over an additive white Gaussian noise channel with \( \text{SNR} = 26 \, \text{dB} \). Let the transmission bandwidth be \( 1 \, \text{Hz} \). According to the Shannon-Hartley law, what is the capacity of this channel in number of bits per second? Round your result to nearest lower integer. You should show process of computation clearly.

13. (6%) Consider two different binary channel codes with the same minimum Hamming distance among codewords. Give one sufficient condition (or one set of sufficient conditions) on the properties of the two codes under which we can get the same coding gain in optimal soft-decision decoding with coherent BPSK transmission over additive white Gaussian noise channels. Justify your answer.