Part I. Algorithms
1. (10%) Consider a set of N distinct real numbers. Can it be done in O(N) time to determine two numbers \( x \) and \( y \) from these \( N \) numbers such that \( |x - y| \) is maximum? If yes, state the algorithm briefly. If no, state your reason and the best possible algorithm.

2. (8%) Consider the following graph algorithm: (a) Divide the graph into two parts and construct the minimum spanning tree for each part. (b) Connect the two parts by the shortest edge connecting the two parts. Does this divide-and-conquer algorithm work for finding minimum spanning tree? Why?

3. (7%) Is it possible to find the longest path in a graph in polynomial time? If yes, state your algorithm and analyze its complexity. If no, state the reason.

4. (10%) Let \( T(n) = \Theta(f(n)) \). Assume that \( T(n) \) is constant for sufficiently small \( n \). Derive \( f(n) \) in the simplest formula for each of the following \( T(n) \).
   
   (a) \( T(n) = 3T(n/2) + n \log n \).
   (b) \( T(n) = 5T(n/5) + n / \log n \).
   (c) \( T(n) = T(3n/4) + T(n/5) + n \).
   (d) \( T(n) = 3T(n/3 + 5) + n / 2 \).
   (e) \( T(n) = n^{1/2}T(n^{1/2}) + n^{1/2} \).

5. (8%) The input is a sequence of \( n \) integers with many duplications, such that the number of distinct integers in the sequence is \( O(\log n) \). (a) Design an \( O(n \log \log n) \) sorting algorithm to sort such sequences (with at most \( O(n \log \log n) \) comparisons in the worst case). (b) Explain why the lower bound \( \Omega(n \log n) \) for sorting is not satisfied in this case.

6. (7%) The input is a set of \( n \) rectangles all of whose edges are parallel to the axes. Design an \( O(n \log n) \) algorithm to mark all the rectangles that are contained in other rectangles.

Part II. Data Structure
7. (5%) Please order the following function by growth rate in increasing order: \( N, \sqrt{N}, N^{1.5}, N^2, \log N, \log(\log N), N \log N, N \log(\log N), N \log^2 N, 2^N, 29, N^2 \log N, \sqrt{N} \).

8. (9%) Given the expression, \((A+B) \times D+E/(F+A \times D)+C\), please
   
   (a) (2%) Give the postfix expression of the expression.
   (b) (3%) List the content of stack after the operand \( F \) is read in postfix transformation.
   (c) (4%) Given that \( A=10, B=20, C=15, D=30, E=2, F=10 \) please list the process in evaluating the postfix expression obtained from (a).

9. (4%) Given the following declarations of the linked list with a header node, please complete the routines of insertion and deletion.
   ```
class DblList;

class DblListNode {
    friend class DblList;
    private:
        int data;
        DblListNode *llink, *rlink;
    };
```
class DblList {
public:
    // List manipulation operations
private:
    DblListNode *first; // points to head node
};

(a) (2%) The routine to deletion from a doubly linked circular list
void DblList::Delete(DblListNode *x) {
    if(x == first) cerr<<"Deletion of head node not permitted"
        <<endl;
    else {
        ::
    }
}

(b) (2%) The routine to insert into a doubly linked circular list
void DblList::Insert(DblListNode *p, DblListNode *x)
    // insert node p to the right of node x
{
    p->link = x;
    ::
}

10. (7%) Tree problems
   (a) (2%) Show array representations and linked list representations of the
       following two binary trees.
   (b) (2%) Describe the distinctions between trees and binary trees.
   (c) (3%) Draw threaded trees of the following two binary trees.

11. (9%) Given a complete graph of vertices K, try to (1) find the number of
    spanning trees (2) find the number of edges (3) prove your answers.

12. (7%) Given a list of n elements and assuming n is larger than 100, compare the
    performance of the following sorting algorithms: (a) heap sort (b) insertion sort
    (c) quick sort (d) merge sort. (Hint: list them in ascending or descending order)

13. (9%) Compare the complexity of operations
    (a) Search an entry
    (b) Delete an entry
    (c) Insert a new entry
    for the following data structures: Linked list, Sequential list and AVL tree.