1. Let $Y_1$ and $Y_2$ have a bivariate normal distribution with mean $E(Y_i) = \mu_i$ and $V(Y_i) = \sigma_i^2$ for $i = 1, 2$, and the covariance is $Cov(Y_1, Y_2) = c$.

(a) Define the joint probability density function of $(Y_1, Y_2)$. (10%)
(b) Find the conditional distribution of $Y_1$ given that $Y_2 = y_2$. (10%)
(c) Assume $\mu_i = 0$, and $\sigma_i^2 = 1$ for $i = 1, 2$, and $c = 0$. Find the probability that $Y_1^2 + Y_2^2 < 1$. (10%)

2. Suppose that $X_1, \ldots, X_n$ is a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$. In testing the population mean $H_0: \mu = \mu_0$ versus $H_0: \mu = \mu_1$, where $\mu_1 = \mu_0 + \delta$ and $\delta > 0$.

(a) Assume $\sigma^2$ is known and a $Z$ test is conducted. Find the sample size required to detect the difference $\delta$ for fixed values of Type I and II errors $\alpha$ and $\beta$. (10%)
(b) Assume $\sigma^2$ is unknown and a $t$ test is conducted. Find the sample size required to detect the difference $\delta$ for fixed values of Type I and II errors $\alpha$ and $\beta$. (10%)

3. Consider the continuous distribution with the probability density function

$$f(x) = \frac{1}{2} \text{ when } 0 \leq x \leq 0 + 2, \text{ and } f(x) = 0 \text{ otherwise.}$$

(a) Set up a test of hypothesis $H_0: \theta = \theta_0 = 3$ for $\alpha = 0.05$. (10%)
(b) Calculate the power in case of the alternative $H_1: \theta = \theta_1 = 4$. (10%)

4. Consider the following layout of a $2 \times 2$ contingency table

<table>
<thead>
<tr>
<th>Row variable</th>
<th>Column variable: Group</th>
<th>1</th>
<th>2</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Successes</td>
<td></td>
<td>$X_1$</td>
<td>$X_2$</td>
<td>$X$</td>
</tr>
<tr>
<td>Failures</td>
<td></td>
<td>$n_1 - X_1$</td>
<td>$n_2 - X_2$</td>
<td>$n - X$</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td>$n_1$</td>
<td>$n_2$</td>
<td>$n$</td>
</tr>
</tbody>
</table>

(a) Perform a $Z$ test for the null hypothesis that there is no difference between two population proportions of success. (10%)
(b) Perform a $\chi^2$ test for the difference between two population proportions of success. (10%)
(c) Show the $\chi^2$ test statistic in (b) is equivalent to the square of the $Z$ test statistic in (a). (10%)