1. (15%) Solve following integral equation:

\[ y(t) = 1 - \sinh t + \int_0^t (1 + \tau) y(t - \tau) d\tau \]

2. (15%) An object of mass \(m\) falling downward experiences the gravity force and the resistance force of the air. Assume that the magnitude of the resistance force is proportional to the speed of the object. The initial speed and position of the object are \(v(0) = v\) and \(y(0) = y\), respectively. Find the position of the object, \(y(t)\).

3. 一維之熱傳問題可用下列方程式及邊界條件來描述

\[
\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, t > 0
\]

\[
\frac{\partial u}{\partial x} (0, t) = \frac{\partial u}{\partial x} (L, t) = 0, \quad t > 0
\]

\[ u(x, 0) = f(x), \quad 0 < x < L \]

其中 \(a\) 及 \(L\) 分別為常數及棒長

a. 此問題如用分離變數法來處理時，其時間及空間函數型態應為何？(15%)
b. 帶入 \(u(x, 0) = f(x), \quad 0 < x < L\) 條件解出解答(10%)

4. 請找出下列複變數級數之收斂半徑 (10%)

\[
\sum_{n=0}^{\infty} \frac{n+1}{2^n} (z + 3i)^n
\]

5. (20%) Find the tangential and normal acceleration of the motion given by

\[
\vec{r}(t) = \vec{i} + \frac{t^2}{2} \vec{j}
\]

6. (15%) If a 3 by 3 matrix \(A\) has eigenvalues 1, 2 and 3. What are the eigenvalues of \((4A^2 + A^2)\)?