1. (15%) Let \( f \) be a one-to-one function defined on an interval \((a, b)\). If \( f \) is differentiable and its derivative does not take on the value 0, prove that \( f^{-1} \) is differentiable and \( (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \).

2. (15%) The ellipse \( r = \frac{8}{4 + 3\cos \theta} \) has right focus at the pole, major axis horizontal. Without resorting to \( xy \)-coordinates, (a) find the eccentricity of the ellipse, (b) locate the ends of the major axis, (c) determine the length of the minor axis, (d) sketch the ellipse.

3. (15%) Prove that if \( \lim_{x \to c} f(x) = l \) and \( \lim_{x \to c} g(x) = m \), then (i) \( \lim_{x \to c} [f(x) + g(x)] = l + m \), (ii) \( \lim_{x \to c} [af(x)] = al \) for each real \( a \), (iii) \( \lim_{x \to c} [f(x)g(x)] = lm \).

4. (10%) A manufacturing plant has a capacity of 25 articles per week. Experience has shown that \( n \) articles per week can be sold at a price of \( p \) dollars each where \( p = 110 - 2n \) and the cost of producing \( n \) articles is \( 600 + 10n + n^2 \) dollars. How many articles should be made each week to give the largest profit.

5. (10%) Let \( m_n = \frac{1}{n} (a_1 + a_2 + \cdots + a_n) \). (i) Prove that if \( \{a_n\} \) is increasing, then \( \{m_n\} \) is increasing. (ii) Prove that if \( a_n \to 0 \), then \( m_n \to 0 \).

6. (15%) On a clock, what time between 2 and 3 o’clock will the two hands (one for hour and another for minute) coincide?

7. (10%) Expand \( g(x) = x^2 \ln x \) in power of \( x - 1 \).

8. (10%) Evaluate \( \int \int_R (y - 2x) \, dx \, dy \) where \( R: 1 \leq x \leq 2, \ 3 \leq y \leq 5 \).