Prob. 1 (25%)

Let \( A \) be the following matrix, compute \( A^{200} \).

\[
A = \begin{bmatrix}
2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2
\end{bmatrix}
\]

Prob. 2 (25%)

If \( F = yi + (x - 3xz)j + xyk \), evaluate \( \int_S (\nabla \times F) \cdot \vec{n} dS \), where \( S \) is the surface of the sphere \( x^2 + y^2 + z^2 = r^2 \) above the \( xy \) plane, \( \vec{n} \) is the normal vector of the surface \( S \).

Prob. 3 (25%)

Solve \( y \) for the following differential equation:

\[
xy' = y + \sqrt{x^2 + y^2},
\]

subjected to

\( y(12) = 5 \)

Prob. 4 (25%)

(a) Solve

\[
y' + 2y = 3u' + 2u, \quad y(0) = 2,
\]

where \( u(t) \) is a unit step function.

(b) Check in the solution of part (a) if \( y(0) = 2 \). If not, try to explain.