1. Let the *Laplace Transform* of a function $f(t)$ be $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$. Use the method of *Laplace Transform* to solve the integral equation, 

$$f(t) = t + \int_0^t f(t - \lambda)\,d\lambda \quad (20 \text{ points})$$

2. Find Fourier series solution of

$$\frac{d^2T}{dx^2} - T = -\delta(x - a), \quad 0 < x < 1$$

$$\frac{dT(0)}{dx} = \frac{dT(1)}{dx} = 0,$$

where $\delta$ is the *Dirac delta* function, $a$ is a constant and $0 < a < 1$. (20 points)

3. Evaluate the following surface integral

$$I = \iint_S \vec{F} \cdot \hat{n} \,dA$$

where $\vec{F} = e^x \hat{i} - ye^x \hat{j} + 3xk$ and $S$ is the surface of $x^2 + y^2 \leq a$, $|z| \leq h$. (20 points)

4. Use the residue theory to evaluate the following improper integral,

$$\int_0^\infty \frac{x \sin x}{x^2 + 4} \,dx \quad (20 \text{ points})$$

5. A matrix, $A$, is called diagonalizable if there exists $Q$ such that $Q^*AQ = D$ is diagonal. Use the *method of diagonalization* by finding $Q$, to solve the following system of differential equations:

$$x' = x + 4y$$

$$y' = x + y$$

where primes denotes $d/dt$. (20 points)