1. (11%) Solve \( x^2(y')^2 - 2(xy - 4)y' + y^2 = 0 \).

2. (11%) Using Laplace transform method, solve

\[
y'' + 9y = \cos 2x, \quad y(0) = 1; y\left(\frac{\pi}{2}\right) = -1.
\]

3. (11%) Using the Residue Theory, evaluate

\[
\int_0^\infty \frac{\cos ax}{x^2 + 1} \, dx, \quad a > 0.
\]

4. Consider the Figure 1 and derive the following formula.
   a. (8%) The equation of a plane through the point \( P_0(x_0, y_0, z_0) \) with normal direction
      \( \mathbf{N} = ai + bj + ck \) is \( a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \).
   b. (8%) The parametric equation on a line \( L \) through a point \( P_0(x_0, y_0, z_0) \) in the
      direction of the vector \( \mathbf{v} = ai + bj + ck \) is \( L(t) = (at + x_0)i + (bt + y_0)j + (ct + z_0)k \).
   c. (8%) The normal vector \( \mathbf{N} \) to a surface \( F(x, y, z) = 0 \) is \( \mathbf{N} = \frac{\partial F}{\partial x}i + \frac{\partial F}{\partial y}j + \frac{\partial F}{\partial z}k \).

![Figure 1](image-url)
5. (10%) Determine the eigenvalues and eigenvectors of

\[
A = \begin{bmatrix}
5 & 2 & 2 \\
3 & 6 & 3 \\
6 & 6 & 9
\end{bmatrix}
\]

6. (15%) Show that the Laplacian in polar coordinates \( r \) and \( \theta \) defined by \( x = r \cos \theta \) and \( y = r \sin \theta \) is

\[
\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}.
\]

7. a. (10%) Using power series to solve

\[
y' + ky = 0
\]

in which \( k \) is constant.

b. (8%) Is the series from (a) equal to \( e^{-ky} \)? Why?