Prob. 1 (17%)

\[ x(t) = a(t) \cos(\omega t + \varphi); \quad u(t) = a(t)e^{\omega t}, \quad j = \sqrt{-1}. \]

What is the relationship between the Fourier transforms of \( x(t) \) and \( u(t) \)?

Prob. 2 (17%)

Solve the following problem.

\[ -a^2 \frac{\partial^2 y}{\partial x^2} + \frac{\partial^3 y}{\partial t^3} = \phi(x) \sin \omega t \]

st.

\[ y(0,t) = y(L,t) = 0; \quad y(x,0) = \frac{\partial y}{\partial t}(x,0) = 0. \]

Prob. 3 (16%)

Calculate Laplace transforms of a real-valued, square-wave function \( f(t) \) with period \( 2 \times c \), where

\[ f(t) = 0 \quad \text{if} \quad t < 0; \]
\[ f(t) = 1 \quad \text{if} \quad nc \leq t \leq (n + 1)c, \quad \text{and} \]
\[ f(t) = -1 \quad \text{if} \quad (n + 1)c < t < (n + 2)c, \quad \text{for} \quad n = 0, 1, 2, 3, \ldots. \]

Prob. 4 (11%)

Let \( A \) be a \( 2 \times 2 \) real and diagonalizable matrix; i.e., \( A \in \mathbb{R}^{2 \times 2} \). If \( A \) has a complex eigenvalue \( \sigma + j\omega \) with its corresponding eigenvector \( V_\sigma + jV'_\sigma \), where \( \sigma, \omega \in \mathbb{R} \), and \( V_\sigma, V'_\sigma \in \mathbb{R}^{2 \times 1} \), show that

a) the matrix \( A \) has the other complex eigenvalue \( \sigma - j\omega \) with a corresponding eigenvector \( V_\sigma - jV'_\sigma \) (5%)

b) the matrix \( A \) is similar to the real matrix \( \bar{A} = \begin{pmatrix} \sigma & \omega \\ -\omega & \sigma \end{pmatrix} \); i.e., there exists a real \( 2 \times 2 \) matrix \( T \) such that \( T^{-1}AT = \bar{A} \). (15%)
Prob. 5 (17%)

Let \( v = rz \hat{e}_r - 3z \hat{e}_\theta + rz^2 \hat{e}_z \), evaluate the surface integral \( \int_S \hat{n} \cdot v \, dA \), including the top, bottom, and side, for a cylinder \( 0 \leq r \leq 2 \), \( 0 \leq z \leq 6 \).

Prob. 6 (16%)

Find the Laplace inverse transform of \( \frac{s}{(s^2 + a^2)^2} \) based on convolution theorem.