Prob. 1 (17%)
Find the half-range cosine expansion and the half-range sine expansion of the function \( f(t) = t^2, 0 \leq t < 1 \). Which has the problem with uniform convergence (explain)?

Prob. 2 (17%)
Find the eigenvalues and the eigenfunctions of the following problem.

\[
\alpha^2 \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial t^2} = 0
\]

st.

\[
y(0, t) = \frac{\partial y}{\partial x}(0, t) = \frac{\partial^2 y}{\partial x^2}(L, t) = \frac{\partial^3 y}{\partial x^3}(L, t) = 0.
\]

You need not find the eigenvalues explicitly. Just write down the characteristic equation which the eigenvalues must satisfy.

Prob. 3 (16%)

Calculate Laplace transforms of a real-valued, square-wave function \( f(t) \) with period \( 2 \times c \), where

\[
f(t) = 0 \quad \text{if} \quad t < 0,
\]

\[
f(t) = 1 \quad \text{if} \quad nc \leq t \leq (n + 1)c, \quad \text{and}
\]

\[
f(t) = -1 \quad \text{if} \quad (n + 1)c < t < (n + 2)c, \quad \text{for} \quad n = 0, 1, 2, 3, \ldots
\]

Prob. 4 (17%)

Let \( A \) be an \( n \times n \) diagonalizable matrix with characteristic equation

\[
\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0 = 0
\]

Prove that

\[
A^n + a_{n-1}A^{n-1} + \cdots + a_1A + a_0I_n = 0_n
\]

(This is a special case of the Cayley-Hamilton theorem, which asserts that a matrix satisfies its own characteristic equation. The theorem is true for any square matrix, but is easier to prove if \( A \) is diagonalizable.)
Prob. 5 (17%)
Let \( v = rz\hat{e}_r + 3\hat{e}_\theta + rz^2\hat{e}_z \), evaluate the surface integral \( \iint_S \hat{n} \cdot vdA \), including the top, bottom, and side, for a cylinder \( 0 \leq r \leq 3 \), \( 0 \leq z \leq 6 \).

Prob. 6 (16%)
Find the Laplace transform of \( \cos at \cdot \frac{\sin at}{a} \) based on convolution theorem.