1. Evaluate the following integrals: (5 points each)
   (a) $\int \frac{x}{x^4 + 6x^2 + 5} \, dx$
   (b) $\int x \frac{dx}{(x - 2)^{3/2}}$
   (c) $\int e^x \sin x \, dx$

2. Evaluate $\int_0^\pi \int_0^{\cos y} x \sin y \, dx \, dy$ (5 points)

3. Find the optimum time to replace a car if rates of depreciation and accumulation of maintenance costs are, respectively,

   $f(t) = -\frac{V}{s_0} (t - 10)$ and $g(t) = \frac{V}{400} t^2$, (10 points)

   where $V$ is the value of the new car. Assume that the car must be replaced by the time it has depreciated to no value.

4. Find $\frac{dy}{dx}$ where $y = x^2$, $x > 0$ (10 points)

5. Use Taylor’s series to determine $a$ and $b$ in the following formula. (10 points)

   $e^{ib} = a + ib$

6. Find $\lim_{x \to \infty} \frac{e^x + e^{-x} - 2}{1 - \cos(2x)}$ (10 points)
7. Let 
\[
A = \begin{bmatrix}
1 & 0 & -4 \\
0 & 5 & 4 \\
-4 & 4 & 3
\end{bmatrix}
\]
(a) Find the eigenvalues and associated eigenvectors of \( A \). (8 points)
(b) Find a nonsingular matrix \( P \) such that \( P^{-1}AP \) is diagonal. Is \( P \) unique? Explain. (4 points)
(c) Find the eigenvalues of \( A^4 \). (4 points)
(d) Find the eigenvalues and associated eigenvectors of \( A^2 \). (4 points)

8. Let \( A \) be an \( n \times n \) matrix with integer entries. Prove that \( A \) is nonsingular and \( A^{-1} \)
has integer entries if and only if \( \det(A) = \pm 1 \). (10 points)

9. Let 
\[
A = \begin{bmatrix}
1 & -2 & 1 & 0 \\
2 & 1 & 1 & 2 \\
1 & -7 & 2 & -2
\end{bmatrix}
\]
Describe the set of all vectors \( b \) in \( \mathbb{R}^3 \) for which the linear system \( Ax = b \) is consistent. (5 points)

10. For an \( n \times n \) matrix \( A \), the trace of \( A \), is defined as the sum of the diagonal entries of \( A \). Prove that the trace defines a linear transformation from \( \mathbb{M}_{n \times n} \) to the vector space of all real numbers. (5 points)