(14%) 1. Consider the water flow sudden contraction of a circular tube as shown below. The pressure difference between Section A and Section B is $P$ KN/m$^2$. Assume that the desired volumetric flow is equal to $Q$ m$^3$/sec and $d_1 = D$ cm. Please derive the equation for determining $d_1$.

(14%) 2. Consider a flow through a circular pipe (radius=$R$) with the velocity profile

$$u = u_{max} (1 - r/R)^a$$

where $u_{max}$ is the maximum velocity at the centerline and $a$ is a constant. Please derive the equation for determining the average velocity.

(8%) 3. A flow is represented by the velocity field

$$\vec{V} = 7x\hat{i} + 13\hat{j} - 7z\hat{k}.$$ 

Determine if the flow is (a) incompressible and (b) irrotational. (Be sure to explain your answer. No credit will be given if no explanation is provided.)

(14%) 4. A cylindrical tank, as shown below, is to be slowly drained out by gravity. The time required to drain the tank completely out is $t$ seconds. Please derive the equation for determining $d$. 

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The text is a question about fluid dynamics, specifically dealing with flow contraction, velocity profile, and cylindrical tank drainage. The text is presented in Chinese and translated into English. The questions involve deriving equations for determining various parameters in fluid dynamics scenarios.
(15 %) 5. The drag force, $\mathbf{F}$, on a stationary object (such as a ball) immersed in a uniform stream depends on the relative velocity between the object and the flow, $\mathbf{U}$, the characteristic dimension of the object, $L$, the fluid density, $\rho$, and the fluid viscosity, $\mu$. Obtain a set of dimensionless groups that can be used to correlate experimental data of $\mathbf{F}$.

(15 %) 6. (i) Determine the force of the water jet in Newton on the inclined plate as a function of the angle $\theta$ as shown below. Consider two-dimensional case and assume that the flow is inviscid. (ii) Determine the relationship between $A_2$ (area of flow section 2) and $A_1$ (area of entering flow section 1); and the relationship between $A_3$ (area of flow section 3) and $A_1$.

$$A_1 = \frac{L}{30} \mathcal{C}_A$$

(11 %) 7. The velocity profile for fully developed turbulent flow through a smooth circular pipe is divided into three regions. Answer the following questions:

(i) (5%) In the region very close to the wall where viscous shear is dominant, what is the name of the region? What is the relationship between the local time average velocity $\bar{u}$ in axial direction and $y$.

(ii) (3%) In the region where both viscous and turbulent shear are important, what is the equation for the velocity profile of $\bar{u}$?

(iii) (3%) In the central region where turbulent shear is dominant, what is the equation for the velocity profile of $\bar{u}$?

Note: symbol $y$ is the distance measured from the wall ($y = R-r$; $R$ is the pipe radius), $u^*$ is the friction velocity, $U$ is the centerline velocity, $\nu$ is the kinematic viscosity of the flow.

(9 %) 8. The region of flow near where the fluid enters the pipe is termed the entrance region.

(i) (6%) What is the dimensionless number that governs the dimensionless entrance length, $L/D$ ($L$: length of the entrance region, $D$: pipe diameter)? Define this dimensionless number.

(ii) (3 %) Beyond the entrance region, does the velocity profile vary with distance along the pipe?