1. (6%) Show that if \( f(n) = a_m n^m + \ldots + a_1 n^1 + a_0 \) and \( a_m > 0 \) then \( f(n) = \Theta(n^m) \), where \( f(n) = \Theta(g(n)) \) iff there exist positive constants \( c_1, c_2, \) and \( n_0 \) such that 
\[
    c_1 g(n) \leq f(n) \leq c_2 g(n) \quad \text{for all } n > n_0.
\]

2. (6%) If an integer array \( MM[m] \) is used to the physical storage of \( n \) integer stacks, propose an abstract data structure for these stacks with a function of inserting an integer into a given stack. A stack is fully only if all spaces of the arrays are used up.

3. (6%) Show the max heaps that are constructed by the following data in the given order: 10, 12, 5, 8, 15, 7

4. (7%) Propose a generalized list for the representation of polynomials in several variables. Show the representation of the following polynomials in your generalized list:

   \[
   t = 2x^3 y^4 + 3x^3 y^4 \\
   u = 4x^3 y^2 z + 6x^3 y^2 + x z
   \]

5. (6%) Write an algorithm to obtain all shortest paths in a graph. What is the computing time of your algorithm?

6. (6%) Write an algorithm for \( O(1) \) space merge. Show your algorithm on the following two sequences of data, \((0, 5, 18, 4, 12, 9)\) and \((1, 6, 10, 3, 11, 2)\), at the end of each phase.

7. (6%) What is a binary search tree? An optimal binary search tree? Write an algorithm to construct an optimal binary search tree.

8. (7%) Write an algorithm to split an AVL tree into two AVL trees such that all identifiers in one tree are \(< x\), and the ones in the other \(> x\).
9. (10%) Let \( f(n) = \Theta(T(n)) \). Derive \( f(n) \) in the simplest formula for each of the following \( T(n) \).
   a. \( T(n) = T(n/2) + n^2; T(c) = c, \text{if } c<2 \).
   b. \( T(n) = 7T(n/4) + n^2; T(c) = c, \text{if } c<2 \).
   c. \( T(n) = 4T(n/4) + n^2; T(c) = c, \text{if } c<2 \).
   d. \( T(n) = 4T(n/2) + n^2 \log n; T(c) = c, \text{if } c<2 \).
   e. \( T(n) = 4T(n/2) + n^2 / \log n; T(c) = c, \text{if } c<2 \).

10. (8%) Briefly describe a linear-time algorithm for the celebrity problem and analyze its time complexity. (Hint: the celebrity problem is: A person is called the celebrity if all the other (n-1) persons know her/him but she/he does not know others.)

11. (7%) Insert a sequence of keys \( \{1, 2, 3, 4, 5, 6, 12, 11, 10, 9, 8, 7\} \) into an empty AVL tree. Show the AVL trees step by step.

12. (11%) 
   (a) State the method to prove a problem is NP-complete. (4%)
   (b) Prove the vertex-cover problem is NP-complete. (7%)

13. (7%) State an algorithm to decompose an connected undirected-graph into biconnected-components.

14. (7%) State an algorithm for fast fourier transform. What is its complexity?