1. A baseband message \( m(t) = A \text{sinc}(2Bt) \) and a carrier signal \( c(t) = \cos 2\pi f_c t \), where \( f_c \gg B \), are used in the following applications.
   
   (a) i. (2%) Find and plot the Fourier transforms of \( m(t) \) and \( c(t) \).
       ii. (3%) Find the Hilbert transforms of \( m(t) \) and \( c(t) \).
   
   (b) i. (3%) Draw the block diagrams of the modulator and demodulator for a double-sideband(DSB) modulation.
       ii. (3%) Find out the complex envelope of this DSB signal.
   
   (c) i. (3%) Draw the block diagrams of the modulator and demodulator for a single-sideband(SSB) modulation.
       ii. (3%) Find the SSB signal.
       iii. (3%) Find out the complex envelope of this SSB signal.
   
   (d) i. (3%) Find the frequency modulated (FM) signal with peak frequency deviation constant \( f_d \).
       ii. (2%) Find the bandwidth of this FM signal.

2. Consider a random process \( x(t) = \sum_{-\infty}^{\infty} a_k \mathbb{I}[(t - kT - \Delta)/T] \), where \( \mathbb{I}[(\cdot)] \) is the rectangular window function with a height of 1 and a width of \( T \), \( a_k \)'s are independent and identically distributed random variables with \( \text{Prob}(a_k = A) = \text{Prob}(a_k = -A) = 0.5 \) for all \( k \), and \( \Delta \) is a random variable uniformly distributed in \( (-\frac{T}{2}, \frac{T}{2}) \) and is independent of all \( a_k \)'s.
   
   (a) (3%) Draw a typical sample function of this random process.
   
   (b) (3%) Derive the mean function of \( x(t) \).
   
   (c) (4%) Derive the autocorrelation function of \( x(t) \) and draw it to the scale.

3. Consider an AM receiver with envelope detection.
   
   (a) (5%) Write down a formula for a received AM signal including the narrowband Gaussian noise process. Explain each term in your formula.
   
   (b) (5%) Use both equation and phasor diagram to prove that when predetection SNR is large, envelope detection has the same postdetection SNR performance as coherent detection.
   
   (c) (5%) Use both equation and phasor diagram to prove that when predetection SNR is small, envelope detection has a threshold effect, i.e., the message signal is totally lost.
4. Let $s_1(t)$ and $s_2(t)$ be two equally-likely binary signals.

(a) (8%) For unipolar signaling, binary 1's are represented by positive value and the binary 0's are represented by a zero level:

$$s_1(t) = +A, \quad 0 < t \leq T \quad \text{(binary 1)}$$
$$s_2(t) = 0, \quad 0 < t \leq T \quad \text{(binary 0)}$$

Draw the optimum receiver’s block diagram for detecting the binary signals in additive white Gaussian noise with double-sided power spectral density $N_0/2$. What is the resulting error probability?

(b) (8%) For OOK signaling,

$$s_1(t) = A \cos \omega_c t, \quad 0 < t \leq T \quad \text{(binary 1)}$$
$$s_2(t) = 0, \quad 0 < t \leq T \quad \text{(binary 0)}$$

Draw the noncoherent receiver’s block diagram for detecting the binary signals in additive white Gaussian noise with double-sided power spectral density $N_0/2$. Assume $A^2T \gg N_0$. What is the error probability if $s_2(t)$ is sent?

5. (a) (3%) Explain the purpose of an equalizer used in a communication system.

(b) (6%) Consider a channel for which the channel pulse response samples are:

$$p_c[-3T] = 0.001, p_c[-2T] = -0.01, p_c[-T] = 0.1, p_c[0] = 1.0,$$
$$p_c[T] = 0.2, p_c[2T] = -0.02, p_c[3T] = 0.05$$

Explain how to find the tap coefficients for a three-tap zero-forcing equalizer. (Exact numerical result is not necessary.)
6. (a) (5%) Assume that the input to a QPSK receiver is

\[ y(t) = A \cdot d_1(t) \cos(w_c t) - A \cdot d_2(t) \sin(w_c t) + u(t) \quad \text{for} \ 0 < t \leq T_s, \]

where \( u(t) \) is an additive white Gaussian noise with double-sided power spectral density \( N_0/2 \). The signals \( d_1(t) \) and \( d_2(t) \) are either \(-1\) or \(+1\) for \( 0 < t \leq T_s \), depending on the information bits to be transmitted. Let the system structure of the QPSK receiver be as shown below.

\[
\begin{array}{c}
\int_0^{T_s} dt \quad \text{sample at } t = T_s \\
\text{cos}(w_c t) \\
\sin(w_c t) \\
\int_0^{T_s} dt \quad \text{at } t = T_s \\
\end{array}
\begin{array}{c}
y(t) \\
v_1(t) \\
v_2(t) \\
V_1 \\
V_2 \\
\pm 1 \\
\pm 1 \\
\end{array}
\]

Also let \( \int_0^{T_s} \sin(2w_c t) dt = \int_0^{T_s} \cos(2w_c t) dt = 0 \). Derive \( V_1 \) and \( V_2 \), and show that they are uncorrelated given \( d_1(t) \) and \( d_2(t) \).

(b) (4%) What is the optimal error probability for detecting signal \( d_1(t) \)? Does the optimal error probability for detecting \( d_1(t) \) perform better than BPSK? Justify your answer.

(c) (4%) What kind of changes will be made on signals \( d_1(t) \) and \( d_2(t) \), if \( y(t) \) now becomes an input to an offset QPSK receiver? Answer the same question for MSK (specifically, type-I MSK).

7. (a) (4%) A source consists of 6 outputs with respective probabilities

\[ [1/2, 1/4, 1/16, 1/16, 1/16, 1/16]. \]

Determine the entropy of the source.

(b) (4%) Construct a binary Huffman code for the source in (a).

(c) (3%) A channel with input \( X \in \{1, 2\} \) and output \( Y \in \{0, 1, 2, 3\} \) is described by the transition probability matrix

\[
[P(Y|X)] = \begin{bmatrix}
P(0|0) & P(0|1) & P(0|2) & P(0|3) \\
P(1|0) & P(1|1) & P(1|2) & P(1|3) \\
P(2|0) & P(2|1) & P(2|2) & P(2|3) \\
0 & 0.25 & 0.5 & 0.25 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Determine the channel capacity.