轉學考工數試題

1. Try to solve the general solution of the ordinary differential equation. (10%)
   \[ my'' + cy' + ky = F_0 \cos wt \]

2. Try to solve the Euler-Cauchy equations. (13%)
   (a) \( x^2 y'' - \frac{1}{2} xy' + \frac{1}{2} y = 0 \)
   (b) \( x^2 y'' + xy' - y = 0 \)
   (c) \( x^2 y'' - xy' + y = 0 \)
   (d) \( (x^2 - x)y'' - xy' + y = 0 \)

3. Solve the initial value problem by using Laplace Transform. (10%)
   \[ y'' + y = 2t \quad \text{where} \quad y(\xi) = \xi, \quad y'(\xi) = 2 - \sqrt{2} \]

4. (i) Find the Laplace transform of the given function. (10%)
   \[ e^{-t} \sin(wt + \theta) \]
   (ii) The symbol \( \mathcal{L} \) stands for the Laplace transform. Show that
   \[ \mathcal{L} \left( \cosh(at) \cos(at) \right) = \frac{s^3}{s^4 + 4a^2} \] (10%)
   (iii) Using Laplace transform, solve the following given initial value problem:
   \[ y'' + 6y + 8y = -2e^{-3t} + 3e^{-5t} \]
   with \( y(0) = 4 \), \( y'(0) = -14 \) (13%)

5. (1) Find all eigenvalues (characteristic values) and eigenvectors (characteristic vectors) of the following matrix. (20%)
   \[
   \begin{pmatrix}
   1 & -3 & 3 \\
   3 & -5 & 3 \\
   6 & -6 & 4
   \end{pmatrix}
   \]

   (2) Solve the following linear equations. (14%)
   \[
   \begin{align*}
   2x + y - z + w &= 5 \\
   x + 2y &= 3 \\
   2x + 2y - z + 3w &= 9 \\
   x + y - z + w &= 4 \\
   2x + 3y + 3w &= 8
   \end{align*}
   \]