Three points for each of the following problems from 1 through 21.

1. Which of the following is not considered as a model in database
   (a) relational model (b) file model (c) network model (d) hierarchical model

2. Software that translates assembly language into machine language is
   (a) binary translator (b) an assembler (c) a compiler (d) a link editor

3. The process of allocating main memory to programs and keeping the programs in memory
   separate from each other is called
   (a) memory protection (b) virtual storage (c) memory management (d) real storage

4. The process of converting from analog to digital is called
   (a) modulation (b) telecommunication (c) line switching (d) demodulation

5. The physical layout of a LAN is called
   (a) topology (b) link (c) contention (d) ATM

6. The study of Digital Signature is a topic in
   (a) data security (b) antivirus (c) firewall (d) worm

7. Which of the following is not correct for JAVA language
   (a) object-oriented language (b) interpreted language (c) compiled language (d) used in internet programming

8. Another name for free software is
   (a) open source software (b) copy protected software (c) public domain software (d) shareware

9. The branch of computer science that explores using computers in tasks that require intelligence, imagination and insight is called
   (a) artificial reality (b) cyberspace explorer (c) Java (d) artificial intelligence

10. How large is one Gigabyte
    (a) 10 megabytes (b) 100 megabytes (c) 1000 megabytes (d) 10000 megabytes

11. Which of the following sorting algorithms can sort n data in O(n log n) worst case time?
    (a) bubble sort (b) quick sort (c) heap sort (d) insertion sort

12. Let G = (V, E) be an undirected graph with n vertices, where n ≥ 1, and M be the
    adjacent matrix of G. Which of the following statements is not correct?
    (a) \( \sum_{j=1}^{n} M(i, j) \) is the degree of vertex i
    (b) \( \sum_{i=1}^{n} \sum_{j=1}^{n} M(i, j) \) is the number of edges of G
    (c) \( \sum_{i=1}^{n} \sum_{j=1}^{n} M(i, j) \) is even.
    (d) \( M \) is a 2-dimensional \( n \times n \) array
13. \( T(n) = 3T\left(\frac{n}{3}\right) + n \). \( T(n) \) is
   (a) \( O(n) \)  (b) \( O(n \log n) \)  (c) \( O(n \log \log n) \)  (d) \( O(n^2) \)

14. Which one of the following statements is incorrect?
   (a) \( n \log n = O(n^2) \)
   (b) \( 6^n \cdot \log n + 1 \cdot n = O(n^2) \)
   (c) \( n^2 = O(n^2) \)
   (d) \( 15n^2 + 100n^2 = O(n^2) \)

15. Store the numbers 30, 12, 9, 8, 6, 2, 3 in a min-heap tree in that order. Which of the following array represents this min-heap tree.

16. Transfer the infix expression \( A B C + D E * A * C \) to the prefix expression.
   Which one is correct?
   (a) \( ABCD * E * AC \)
   (b) \( ABCD + E * AC \)
   (c) \( ABCD * E * AC \)
   (d) \( ABCD * E * AC \)

17. The following traversals unambiguously define a binary tree. What is its preorder traversal?
   Inorder traversal: ABHCDEGF
   Postorder traversal: ABCDEHGF
   (a) ABCDEFGHI
   (b) ABCDEHIGF
   (c) FGHBABCDE
   (d) ABCDEHGF

18. Consider a selection tree with \( k \) runs. What is the number of levels in the tree?
   (a) \( \lceil \log_k k \rceil \)
   (b) \( \lceil \log_k k \rceil + 1 \)
   (c) \( k \)
   (d) \( k^2 \)

19. Let \( G = (V, E) \) be an undirected graph with \( n \) vertices and \( e \) edges. For the minimum spanning tree problem, which of the following statements is not correct?
   (a) Kruskal's algorithm is a greedy method.
   (b) The time complexity of Kruskal's algorithm is \( O(e \log e) \).
   (c) Prim's algorithm is a greedy method.
   (d) The time complexity of Prim's algorithm is \( O(e \log e) \).
20. Consider a binary tree, $T$. What is the maximum number of nodes in $T$ of depth $k$?
(a) $2^k$
(b) $2^{k+1}$
(c) $2^{k-1}$
(d) $2^{k+1}$

21. Given a 2-3 tree, $T$, as follows. What is the 2-3 tree after insertion 30 in $T$?

(a)

(b)

(c)

(d)
22. Show that "for any nonempty binary tree, if $n_0$ is the number of leaf nodes and $n_1$ is the number of nodes of degree 2, then $n_0 - n_1 = -1"$. 8p

23. Consider the Union-Find Operation for the problem "set representation". If we apply the weighting rule when we union two sets. Show that the height of the resulting tree is at most $\log_2 (n_1) + 1$. 8p

24. Show the correctness of the LSD radix sort algorithm. 8p

25. "畫象"的遊戲有將來規劃，例如有一個人去買東西，如下圖：如果有四個人拿一個物品出去，就畫四個圖案線，然後在圖案的角落標記任意加入水平線（假設沒有再接續其他線）。畫象的遊戲規則是每個人都選一個起點，隨著垂直線下去，每碰到一個水平線就轉（當然要碰到垂直線時就要再接續往下走）。這遊戲可以畫來標識是因為其必是一到一、也就是說不同的點一定會到達不同的點。試用數學方法說明不論多少人玩這遊戲（也就是不論有多少條垂直線），不論有多少條水平線，不同起點必到達不同終點（必定是一對一）。 12p

![Diagram](https://example.com/diagram.png)